National Center and State Collaborative

NCSC Curriculum Resource to Prepare Students for AA-AAS

Mathematics Content: Measurement and Geometry

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The purposes of the Curriculum Resource Guides are:
• To provide guidance for teaching the Florida Standards to students with Significant Cognitive Disabilities (SWSCD) that both aligns with these standards and provides differentiation for individual student needs • To provide examples for differentiating instruction for a wide range of SWSCD. These examples can be used in planning specific lessons, alternate assessment items, and professional development.

1a. What is "perimeter" and how is it taught in general education settings?

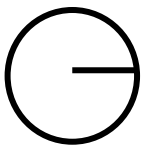
1a.1 Essential knowledge in this content area

The concept of perimeter refers to the distance around a polygon. (A polygon is a shape which is formed by line segments enclosing an area). The distance can be found by adding the lengths of all sides. Students first begin to explore this concept by laying a ruler around all of the sides of an object and then adding all of the lengths. Students can be encouraged to generalize perimeter into the following equation:

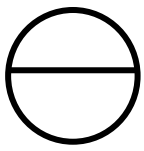
- For rectangles:
  - P = 2l + 2w (where P = perimeter, l = length, and w = width)
- For triangles:
  - Add all sides or P = a + b + c (where P = perimeter and a, b, c = the sides of the triangle)

For circles:

- It is called circumference (the distance around a circle). To find circumference students must know: radius (connects the center to any given point on the circle) or diameter (connects two points on the circle and passes through the center).



radius



diameter

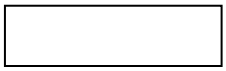
- The ratio of the circumference to the diameter ($^{C}/_{d}$) of any circle is the same for all circles. This ratio is called pi, or π. You can use this relationship to find a formula for circumference. Pi (π) is an irrational number that is often approximated by the rational number 3.14.
- The circumference (C) of a circle is π times the diameter (d), or 2π times the radius (r).
  - C = πd OR C=2πr

1a.2 Common misunderstandings in this content area

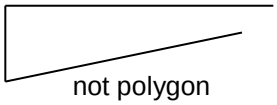
Students may have a hard time understanding the dual meaning of the word length. It is not only the distance measured of each line segment, but it also means the longer side of a rectangle (where the width means the shorter side of the rectangle). In addition, once area is introduced students may become confused between area and perimeter. A helpful analogy to use is that perimeter is the "fence" and area is the "lawn."

1a.3 Prior knowledge/skills needed (can be taught concurrently)

- Identify a polygon



polygon



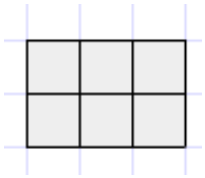
not polygon

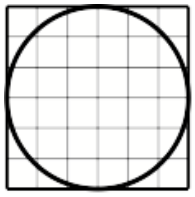
- Addition
- Draw or connect line segments that touch end to end to enclose an area
- Multiply by 2

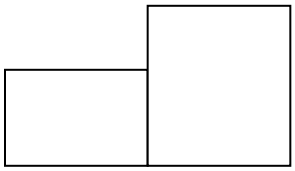
1.b What is "area" and how is it taught in general education settings?

1b.1 Essential knowledge in this content area

The concept of area focuses on determining the amount of space inside of a two-dimensional figure. This is typically done in elementary school with the process of tiling an entire region with a small square piece called a unit square. For example, if we took 1 cm by 1 cm square tiles and covered a rectangle with 6 tiles, we would say that the area of the rectangle is 6 square centimeters.







Also provide opportunities for students to decompose rectilinear shapes to determine area.

Students should have ample hands-on experiences with tiles to construct understanding that calculating area is the process of repeatedly tiling a two-dimensional shape with a unit square. Whether the units are centimeters, inches or other measures does not matter, as long as the unit is consistently used to measure an object. As students

master understanding of the concept you can teach the following formula to find the area of a rectilinear (composed of right [90º] angles) figure:

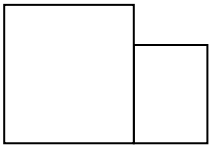
- A = l x w (where A = area, l = length, and w = width)

Area of a circle is more complex. Students can estimate area of a circle by counting squares within, similar to rectangles. The area A of a circle is π times the square of the radius r.

- $A = \pi r^2$

1b.2 Common misunderstandings in this content area

The most common misunderstanding is confusing the concepts of area and perimeter. The perimeter of a shape is the distance around, while the area is the amount of space inside. If teachers move too quickly to a formula to calculate these two concepts, then students become easily confused.



A composite shape is a flat shape composed of two different sized rectangles. Students must break the shape up into rectangles in order to determine the entire area.

1b.3 Prior knowledge/skills needed (can be taught concurrently)

In general education, the student typically will need to:

- Lay tiles on a surface with no gaps or overlaps
- Count tiles after covering an object
- Draw squares on a piece of grid (graph) paper OR count squares on a piece of paper
- Multiply numbers
- Understand concepts of length and width

1c. What is "volume" and how is it taught in general education settings?

1c.1 Essential knowledge in this content area

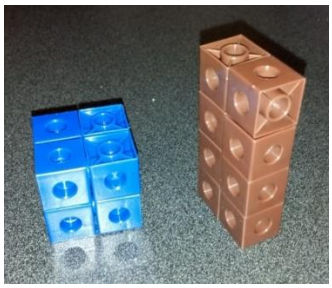
Volume is the amount of space a three dimensional (3-D) figure takes up. Examples of 3-D figures that take up space include rectangular prisms, spheres, cubes, cones, cylinders, and pyramids. Students should understand figures that have volume are 3-D rather than flat, two-dimensional (2-D) objects. Rectangular prisms and cubes have three dimensions: length, width, and height. A flat surface of a 3-D figure is a face. An edge is where two faces meet. A vertex is where the figure comes to a point. The base is the shape used to classify the figure. Students should be provided with numerous hands-on opportunities to explore items that take up space and label the attributes of the figures.

The units used to measure volume are called cubic units. Previously, students used tiles to measure area, and now they will be using cubes to measure volume. Volume is an extension of area. To determine area, students tiled a flat surface with unit squares. To determine the volume of a rectangular prism or cube, students begin by filling the object with unit cubes. The Florida Standards for Math refer to rectangular prisms and cubes as rectilinear shapes, since the shapes are composed of all right angles, and every face can be composed of 1 or more rectangles. Again, whether the units of the cubes are centimeters, inches, or other measures does not matter, as long as the unit is consistently used to measure an object. Students can practice measuring volume by using cubes to make a replica of a given shape and then count the cubes to measure the volume. They can also begin to identify the length, width, and height of the object and its numerical value. As students master understanding of the concept you can teach the following formulas:

- Rectangular Prisms
  - The volume of a prism is the area of the base (B) [length times width] times the height (h).
  - $V = l \times w \times h$ OR $V = B \times h$
- Cylinders
  - The volume of a cylinder is the area of the base (B) [πr^2] times the height (h).
  - $V = B \times h$ OR $V = \pi r^2 \times h$
- Spheres
  - The volume of a sphere is $^4/_3\pi$ times the cube of the radius (r).
  - $V = {}^4/_3\pi \times r^3$

1c.2 Common misunderstandings in this content area

Students may think that if items have the same volume, they must be the same shapes (i.e., “This square has 8 cubic units of volume, therefore all objects with 8 units of cubic volume are squares.”)



Example of two different shapes with the same volume.

1c.3 Prior knowledge/skills needed (can be taught concurrently)

In general education, the student typically will need to:

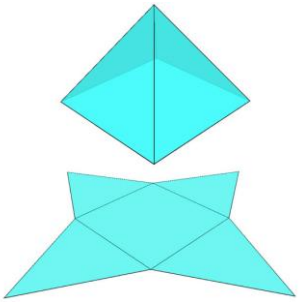
- Identify dimensions of length, width, and height

- Calculating area using a formula
- Count cubes
- Use cubes to fill with no gaps or overlaps
- Experience multiplying numbers
- Understand how to cube a number (e.g., 2^3)

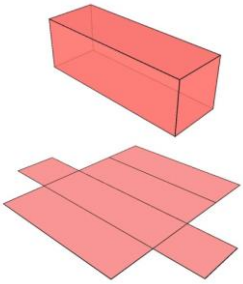
1d. What is "surface area" and how is it taught in general education settings?

1d.1 Essential knowledge in this content area

Surface area is the sum of all the faces of an object. Students can first learn to add the areas of the faces to find the surface area. Nets will help with this. See: http://illuminations.nctm.org/LessonDetail.aspx?id=L570 for further information and examples. Students should be encouraged to engage in activities using nets to deepen their understanding of surface area (e.g., matching net to corresponding 3-D object).



pyramid and net



rectangular prism and net

Then the following formulas can be taught to find the surface area of rectangular prisms, cubes, and square pyramids:

- Rectangular prisms: SA = 2lw + 2lh + 2wh (where SA = surface area, l = length, w = width, and h = height)
- Cubes: SA = $6s^2$ (where SA = surface area, and s = length of sides)
- Square pyramid SA = B + ½ Pl (where SA = surface area, B = base area, P = perimeter of base, l = slant height

1d.2 Common misunderstandings in this content area

Students may believe that they have identified surface area by merely counting all the faces of the object.

1d.3 Prior knowledge/skills needed (can be taught concurrently)

In general education, the student typically will need to:

- Understand the concept of the "face" of objects
- Count the number of "faces" on an object
- Solve equations using order of operations

- Count cubes
- Identify cubes and rectangular prisms
- Understand how to square a number (e.g. 3^2)

1e. What is "Classifying and Comparing Figures" and how is it taught in general education settings?

1e.1 Essential knowledge in this content area

Students should learn to identify 2-D and 3-D figures including: rectangles, squares, triangles, trapezoids, quadrilaterals, pentagons, hexagons, triangles, prisms, cones, cylinders, pyramids, cubes, and spheres. Given pattern blocks, students should be able to compose identified figures. They need to know the salient (defining) and non-salient (non-defining) features of figures. For instance, the defining features of rectangles are: they have four sides, they are closed figures (all lines touch end to end), opposite ends are parallel, and all four angles enclosed are right angles. Non-defining features of a rectangle would be its size, color, or orientation. Knowing the defining features of figures will enable students to compare figures based on attributes.

Attributes of figures students will need to know include:

- Recognize different types of angles:

right acute obtuse

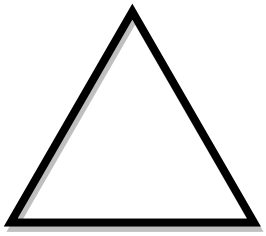
- Understand parallelism and perpendicularity

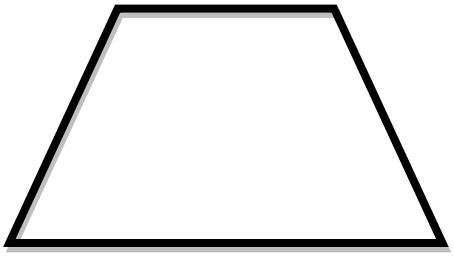
Two examples of parallel lines Two examples of perpendicular lines

Students need to understand congruency. Figures that are congruent have the same shape and size.

Example of congruent shapes Example of non-congruent shapes

Once students understand defining attributes, given the following shapes students should be able to classify them by number of angles, number of sides, number of angles which are greater than ninety degrees, tell whether the shapes are congruent, identify which shape is a quadrilateral, etc.





1e.2 Common misunderstandings in this content area

Students may learn the label of a figure such as “square” and when asked if it is a quadrilateral say “no.” However, since the definition of a quadrilateral figure is that it has four sides then a square is a quadrilateral. Therefore students will need practice sorting shapes into categories. Also, students may have difficulty determining whether shapes are congruent when they are rotated (see example on previous page) and may need repeated practice with determining congruency of shapes with various rotations.

1e.3 Prior knowledge/skills needed (can be taught concurrently)

- Label 2-D and 3-D figures based on attributes (e.g., triangle, pyramid)
- Count (number of angles, sides, etc.)
- Understand same and different

1f. What is “Converting Units of Measurement” and how is it taught in general education settings?

1f.1 Essential knowledge in this content area

In order to be able to convert measurements, students will need to understand that objects can be measured to find length, weight, and capacity (of liquids). Students should know the units of measurement and the relationship between units of measurement in the same system. For example:

Length			
U.S. Unit	**Equivalent**	**Metric Unit**	**Equivalent**
1 foot (ft)	12 inches (in)	1 centimeter (cm)	10 millimeters (mm)
1 yard (yd)	3 feet (ft)	1 meter (m)	100 centimeters (cm)
1 mile (mi)	1,760 yards (yd)	1 kilometer (km)	1,000 meters (m)

1f.2 Common misunderstandings in this content area

Students may find converting standard measures particularly difficult because: (a) the number to multiply or divide to convert will vary based on the given unit (e.g., feet to inches, multiply by 12; yards to feet, multiply by 3); (b) if the conversion does not equal an exact whole number unit of measurement then students may need to combine two

units to provide the answer (e.g., 5 feet, 11 inches) [This is only true with standard measure, with metric measurements decimals may be used.]; and (c) If conversions are more than one unit removed students will be required to perform multiple steps to find the correct amount (e.g., if converting weeks to minutes, students must convert weeks to days, then days to hours, finally hours to minutes).

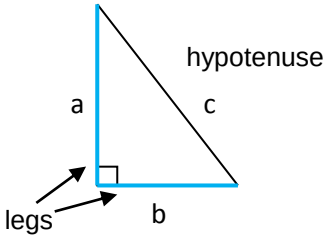
1f.3 Prior knowledge/skills needed (can be taught concurrently)

- Knowledge of both metric and standard units of measurement
- Knowledge of measurement units for length, volume, mass, time
- Knowledge of equivalents (e.g., 1 foot = 12 inches)
  - Students can be provided with a conversion chart if needed
- Multiplication and division
- Number identification
- Use of measurement tools (i.e., line up a ruler's edge to the edge of the object being measured)
- Knowledge of fractions and decimals

1g.What is "Pythagorean Theorem" and how is it taught in general education settings?

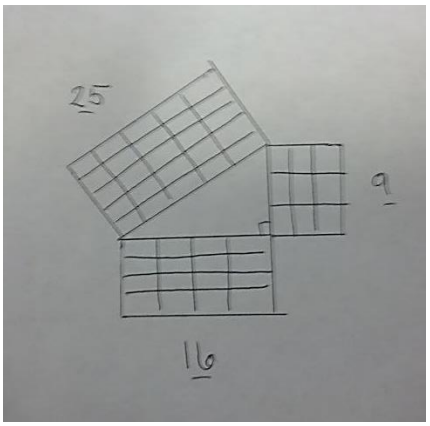
1g.1 Essential knowledge in this content area

Right triangles have 1 right angle and 2 acute angles. The side opposite the right angle is called the hypotenuse and the other two sides are called legs. In any right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse – this is called the Pythagorean Theorem.



$$a^2 + b^2 = c^2$$

Below is a visual model of the concept of the Pythagorean theorem where the length of a = 3, b = 4, c = 5.



$a^2 + b^2 = c^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

1g.2 Common misunderstandings in this content area

Students may forget to use squares to determine length of sides (e.g., compute a + b to find c instead of $a^2 + b^2$ to find c^2). In addition, students may overgeneralize if not given enough practice examples. For example, if given an example such as $3^2 + 4^2 = 5^2$, student may think that $6^2 + 7^2 = 8^2$.

1g.3 Prior knowledge/skills needed (can be taught concurrently)

- Identify right triangles
- Number identification
- Multiplication and Division
- Knowledge of exponents, specifically squaring (x^2) and finding the square root ($\sqrt{}$)

2. What are some of the types of activities general educators will use to teach this skill?

2.1 Activities from General Education Resources

- 6 Trace around a book and estimate then measure the perimeter of the book first with paper clips, then toothpicks, then inches.[1]
- 4 On centimeter grid paper, draw and label a rectangle with a length of 6 centimeters and a width of 2 centimeters. Find the perimeter of this rectangle. Now find the perimeter of the following rectangles (l=12, w=4; l=18, w=6; l=24, w=8). Make a conjecture: How is the perimeter affected if the length and width are changed proportionally?[2]

[2] Greenes, C. E. (2005). *Houghton Mifflin math: Grade 5*. Boston, MA: Houghton Mifflin Co.
[3] Bailey, R., Day, R., Frey, P., Howard, A. C., Hutchens, D. T., McClain, K., Moore-Harris, B. Glencoe/McGraw-Hill. (2004). *Mathematics: Applications and concepts*. New York: Glencoe/McGraw-Hill.

-  14 Given a choice of classroom objects, choose an object, determine which tool is needed to measure perimeter, indicate the formula needed to determine perimeter, then find the perimeter.[3]
-  5 On a map with a scale, lay a string around the state of Florida. Mark the string where it meets itself. Use a ruler to measure the string in centimeters. Use the map scale to estimate the perimeter in miles.[4]
-  6 On 1-inch graph paper, trace around your shoe. Count the number of whole square units covered by the shoe. Count two partially covered squares as one unit. What is the area of the bottom of your shoe?[5]
-  4 Using a geoboard and a rubber band find the area of a variety of shapes.[6]
-  4 On grid paper, draw then cut out a rectangle. Cut a triangle from one side of the rectangle and move it to the other side to form a parallelogram. How does a parallelogram relate to a rectangle? What part of the parallelogram corresponds to the length of the rectangle? What part corresponds to the rectangle's width? Make a conjecture: What is the formula for the area of a parallelogram?[7]
-  14 A gardener has 24 feet of fencing for a garden. What are the dimensions of the rectangle that will enclose the greatest area? (Provide 24 paperclips to represent the 24 feet of fencing.)[8]
-  3 Using a small box and unit cubes, first estimate how many cubes it will take to fill the box. Then place the cubes in the box and count them to determine the volume. Now use the cubes to build a figure. Is the volume the same even though the shape is different? Why or why not?[9]
-  7 Using centimeter cubes, build five different prisms. For each prism, record the dimensions and the number of cubes used (# of cubes, height, length of base, width of base, area of base). Examine the rows of the table. What patterns do you notice? Describe the relationship between the number of cubes needed and the dimensions of the prism.[10]

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[4] Bailey, R., Day, R., Frey, P., Howard, A. C., Hutchens, D. T., McClain, K., Moore-Harris, B. Glencoe/McGraw-Hill. (2004). *Mathematics: Applications and concepts*. New York: Glencoe/McGraw-Hill.
[5] Malestsky, E. M., & Andrews, A. G. (2004). *Harcourt math: Grade 4*. Orlando: Harcourt School Publishers.
[6] Greenes, C. E., Stiff, L. (2005). *Houghton Mifflin math: Grade 6*. Boston, MA: Houghton Mifflin Co.
[6] Greenes, C. E. (2005). *Houghton Mifflin math: Grade 5*. Boston, MA: Houghton Mifflin Co.
[7] Bailey, R., Day, R., Frey, P., Howard, A. C., Hutchens, D. T., McClain, K., Moore-Harris, B. Glencoe/McGraw-Hill. (2004). *Mathematics: Applications and concepts*. New York: Glencoe/McGraw-Hill.
[8] Bennett, J. M., & Holt, Rinehart, and Winston, inc. (2004). *Holt pre-algebra*. Orlando, Fla: Holt, Rinehart and Winston.
[9] Greenes, C. E. (2005). *Houghton Mifflin math: Grade 5*. Boston, MA: Houghton Mifflin Co.
[10] Bailey, R., Day, R., Frey, P., Howard, A. C., Hutchens, D. T., McClain, K., Moore-Harris, B. Glencoe/McGraw-Hill. (2004). *Mathematics: Applications and concepts*. New York: Glencoe/McGraw-Hill.

2.2 Links Across Content Areas

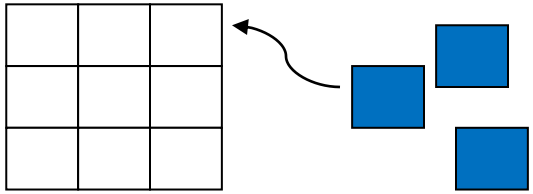
-  In geography or world history, students may need to apply measurement skills with maps to determine distance between locations.
-  Science requires a variety of measurement and geometry skills such as:
    - Measuring out certain volumes to conduct chemistry experiments
    - Finding the mass of organs when dissecting various organisms
    - Determining the rate of water evaporation when the surface of the water has a large area versus a small area
    - Determining whether the attributes of a figure impact its velocity

3. What Florida Standards Access Points Are Addressed in Teaching "Measurement and Geometry"?

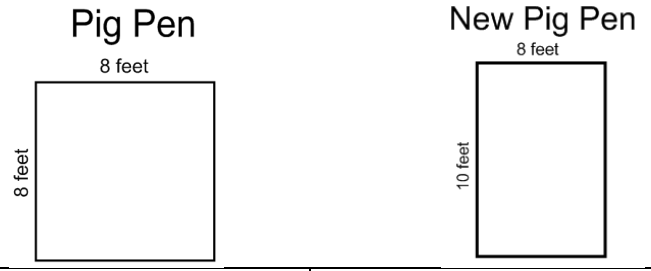
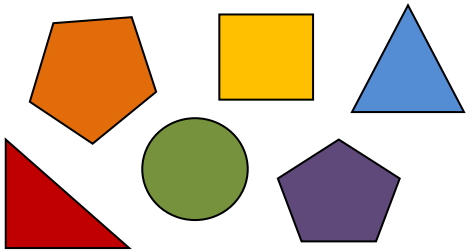
Grade Differentiation	Access Points
3rd grade	MAFS.3.MD.1.AP.1a Solve word problems involving the addition and subtraction of time intervals of whole hours or within an hour (whole hours: 5:00 to 8:00, within hours: 7:15 to 7:45) on a number line. MAFS.3.MD.3.AP.7a Use tiling and repeated addition to determine area. MAFS.3.MD.3.AP.6a Measure area of rectangles by counting unit squares. MAFS.3.MD.1.AP.2b Select appropriate units for measurement involving liquid volume and mass. MAFS.3.MD.1.AP.2c Add to solve one-step word problems involving liquid volume and mass. MAFS.3.MD.1.AP.2a Select the appropriate tool for the measurement of liquid volume and mass. MAFS.3.MD.2.AP.4a Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. MAFS.3.MD.4.AP.8a Use addition to find the perimeter of a rectangle. MAFS.3.MD.1.AP.2d Estimate liquid volume and mass. MAFS.3.G.1.AP.2a Partition a rectangle into equal parts with equal area.
4th grade	MAFS.4.MD.1.AP.3a Solve word problems involving perimeter and area of rectangles using specific visualizations/drawings and numbers. MAFS.4.MD.3.AP.5a Identify an angle in a two-dimensional figure. MAFS.4.MD.3.AP.6a Sketch angles of specific measures. MAFS.4.MD.3.AP.7a Find sums of angles that show a ray (adjacent angles). MAFS.4.MD.1.AP.1a Within a system of measurement, identify the number of smaller units in the next larger unit. MAFS.4.MD.1.AP.1a Complete a conversion table for length and mass within a single system. MAFS.4.MD.1.AP.3a Solve word problems involving perimeter and area of rectangles using specific visualizations/drawings and numbers. MAFS.4.G.1.AP.1c Identify an angle in a two-dimensional figure. MAFS.4.G.1.AP.2a Identify and sort objects based on parallelism, perpendicularity, and angle type. MAFS.4.G.1.AP.3a Identify figures that have a line of symmetry.
5th grade	MAFS.5.MD.1.AP.1a Convert standard measurements of time to solve real-world problems. MAFS.5.MD.1.AP.1b Convert standard measurements of length to solve real-world problems. MAFS.5.MD.1.AP.1c Convert standard measurements of mass to solve real-world problems. MAFS.5.MD.3.AP.4a Determine the volume of a rectangular prism built by "unit cubes." MAFS.5.MD.3.AP.5c Connect the layers to the dimensions and multiply to find the volume of the rectangular prism. MAFS.5.G.2.AP.3a Recognize properties of simple plane figures using polygon-shaped manipulatives.5.GM.1b1 Distinguish plane figures by their properties

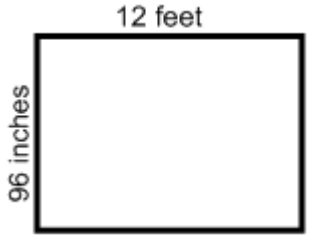
Grade Differentiation	Access Points
	MAFS.5.G.1.AP.1a Locate the x- and y-axis on a coordinate plane. MAFS.5.G.1.AP.1b Locate points on a coordinate plane. MAFS.5.G.1.AP.1c Graph ordered pairs (coordinates).
6th grade	MAFS.6.RP.1.AP.3c Solve one-step real-world measurement problems involving whole number unit rates when given the unit rate ("Three inches of snow falls per hour, how much falls in six hours?"). MAFS.6.RP.1.AP.3a Use ratios and reasoning to solve real-world mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). MAFS.6.RP.1.AP.3b Solve unit rate problems involving unit pricing using whole numbers. MAFS.6.G.1.AP.1b Decompose complex shapes (polygon, trapezoid, and pentagon) into simple shapes (rectangles, squares, triangles) to measure area. MAFS.6.G.1.AP.1a Compose rectangles to find areas of right triangles using graph paper. MAFS.6.G.1.AP.1c Find the area of quadrilaterals using models.
7th grade	MAFS.7.G.2.AP.6b Solve one-step, real-world measurement problems involving area, volume, or surface area of two- and three-dimensional objects. MAFS.7.G.2.AP.4b Measure the circumference of a circle using string. MAFS.7.G.1.AP.1b Draw a scale drawing of a real-world two-dimensional polygon on graph paper. MAFS.7.RP.1.AP.1a Solve one-step problems involving unit rates associated with ratios of fractions. MAFS.7.G.1.AP.2a Construct or draw plane figures using properties. MAFS.7.G.2.AP.6a Add the area of each face of a prism to find the surface area of three-dimensional objects. MAFS.7.G.2.AP.5d Identify vertical angles using visual models and find their measures.
8th grade	MAFS.8.G.1.AP.4c Compare area and volume of similar figures. MAFS.8.G.3.AP.9a Using a calculator, apply the formula to find the volume of three-dimensional shapes (i.e., cubes, spheres, and cylinders). MAFS.8.G.2.AP.8a Apply the Pythagorean theorem to determine lengths/distances between two points in a coordinate system by forming right triangles. MAFS.8.G.1.AP.1a Perform rotations, reflections, and translations using pattern blocks. MAFS.8.G.1.AP.3b Given two figures on a coordinate plane, identify if the image is dilated, translated, rotated, or reflected. MAFS.8.G.1.AP.4a Recognize congruent and similar figures. MAFS.8.G.1.AP.5a Use angle relationships to find the value of a missing angle. MAFS.8.G.2.AP.7a Find the hypotenuse of a two-dimensional right triangle using the Pythagorean theorem. MAFS.8.G.2.AP.6a Measure the lengths of the sides of multiple right triangles to determine a relationship.

Grade Differentiation	Access Points
Grades 9-12	MAFS.912.N-Q.1.AP.1c Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context. MAFS.912.N-Q.1.AP.1d When solving a multi-step problem, use units to evaluate the appropriateness of the solution. MAFS.912.G-MG.1.AP.1a Describe the relationship between the attributes of a figure and the changes in the area or volume when one attribute is changed. MAFS.912.A-REI.2.AP.3a Solve linear equations in one variable, including coefficients represented by letters. MAFS.912.N-Q.1.AP.3a Describe the accuracy of measurement when reporting quantities (you can lessen your limitations by measuring precisely). MAFS.912.G-SRT.1.AP.2a Determine if two figures are similar. MAFS.912.G-SRT.1.AP.2b Given two figures, determine whether they are similar and explain their similarity based on the equality of corresponding angles and the proportionality of corresponding sides. MAFS.912.G-C.2.AP.5c Apply the formula to the area of a sector (e.g., area of a slice of pie). MAFS.912.G-MG.1.AP.3a Apply the formula of geometric figures to solve design problems (e.g., designing an object or structure to satisfy physical restraints or minimize cost). MAFS.912.G-CO.2.AP.7a Use definitions to demonstrate congruency and similarity in figures. MAFS.912.G-CO.1.AP.5a Transform a geometric figure given a rotation, reflection, or translation using graph paper, tracing paper, or geometric software. MAFS.912.G-CO.3.AP.10a Measure the angles and sides of equilateral, isosceles, and scalene triangles to establish facts about triangles.

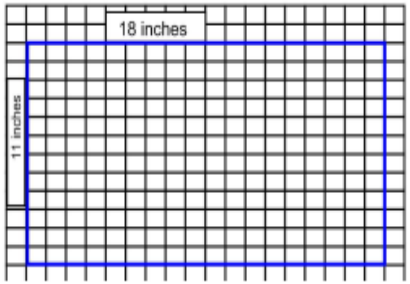
Performance Examples for Priority Access Points			
Grade 3			
Access Point	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Geometry: MAFS.3.G.1.AP.2a Partition a rectangle into equal parts with equal area.	A) Selects the square that is divided into 4 equal parts Choice Selection:  B) Answers: How do you know that these parts *have the same area*? (The response provided by the test example is that the square has the same number of parts in it. Again, this is not technically correct because one could divide the square into four parts of different sizes.)	**Concrete Understandings:** • Understand the concept of equal parts (e.g., fold rectangular pieces of paper into two or four equal pieces). • Partition with concrete objects. • Find the rectangle that is the same or match two congruent rectangles.	**Representation:** • Partition rectangles into two, three, or four equal shares. • Understand the following concepts and vocabulary: equal, partition, area, rectangle, halves, thirds, half of, a third of.
Measurement: MAFS.3.MD.3.AP.6a Measure area of rectangles by counting unit squares.	Student tiles and indicates area of rectangle. 	**Concrete Understandings:** • Count up to 20 objects. • Identify the area on a surface (e.g., piece of paper). • Recognize that area can be determined by covering a rectangular area with square tiles that have no gaps or overlaps. • Use square tiles to cover a rectangle. • Count the number of tiles to determine the area. • Decompose rectangles within a rectilinear figure.	**Representation:** • Select the numeric symbol that represents the number of squares used to find area of a figure. • Count to find the area of a rectangle when given a picture or array.

<table>
<tr><td colspan="4">Grade 4</td></tr>
<tr><td>Access Point</td><td colspan="2">Performance Example</td><td>Essential Understandings: Concrete Understandings and Representations</td></tr>
<tr><td>Measurement
MAFS.4.MD.1.AP.3a
Solve word problems involving perimeter and area of rectangles using specific visualizations/ drawings and numbers.</td><td colspan="2">Given a perimeter that changes, the student will indicate how much more the new perimeter is than the original perimeter.

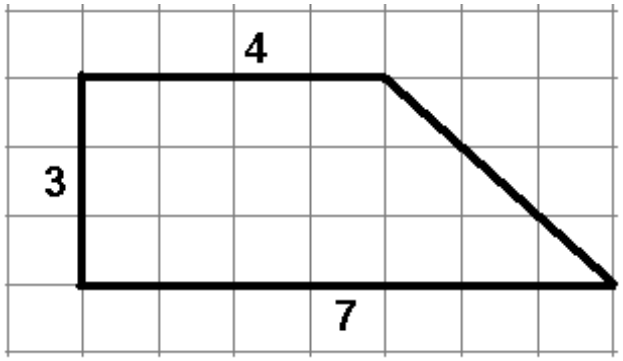
The farmer’s pig pen is 8ft by 8ft. It has a fence around it. The farmer bought a new pig and wants to make the pen bigger. It will be 8ft by 10ft. How much more fence will he need?
 
</td><td>Concrete Understandings:
• Identify the perimeter.
• Identify the area.
• Decompose a rectilinear figure into rectangles.</td><td>Representation:
• Understand the following concepts and vocabulary (pictures/symbols): area, perimeter, length, width, side, +, -, X, ÷.</td></tr>
<tr><td>Geometry:
MAFS.4.G.1.AP.2a
Identify and sort objects based on parallelism, perpendicularity, and angle type.</td><td>Give the students the following shapes:
  </td><td>Sort the shapes into the table. Put the shapes that have three sides on this side (point to column) and the shapes that have five sides on this column (point to the column). Some shapes won’t go on either side.
<table><tr><td>Three sides (3)</td><td>Five sides (5)</td></tr><tr><td></td><td></td></tr></table></td><td>Concrete Understandings:
• Identify attributes within a two-dimensional figure (e.g., rectangles have sides: student identifies sides of rectangle- and angles, student identifies angles in rectangle).</td><td>Representation:
• Identify parallel and perpendicular lines within two-dimensional shapes.
• Recognize and identify right angles.
• Understand the following concepts and vocabulary: face, edge, corner, side, angle.</td></tr>
</table>

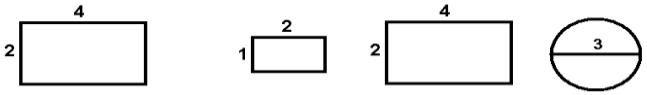
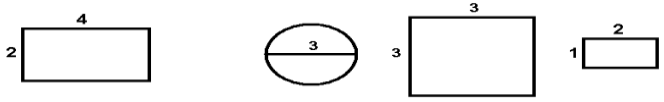
Grade 5			
Access Point	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Measurement MAFS.5.MD.1.AP.1b Convert standard measurements of length to solve real-world problems.	Student converts inches to feet. Cirilla has a pet snake. It is 30 inches long. There are 12 inches in a foot. How many feet long is Cirilla's snake? Snake = 30 inches 12 inches = 1 foot	**Concrete Understandings:** • Recognize that in the same system, I can measure the same object with two different units—height of a desk in both inches and feet.	**Representation:** • Understand concepts and vocabulary: conversion, inch, foot, yard. • Understand standard units and abbreviations (e.g., feet = ft).
Measurement	Student finds area when given measurements in two different units of measure. Henri wants to paint a wall in his bedroom. He needs to know the area of the wall to be sure he buys enough paint. The width of the wall is 96 inches. The length is 12 feet. There are 12 inches in a foot. Remember area equals length times width. What is the area of Henri's wall? 12 inches = 1 foot  What is the area of the wall?		

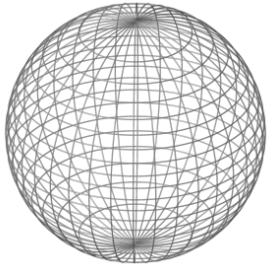
<table>
<tr><td colspan="4">Grade 6</td></tr>
<tr><td>Access Point</td><td>Performance Example</td><td colspan="2">Essential Understandings: Concrete Understandings and Representations</td></tr>
<tr><td>Geometry
MAFS.6.G.1.AP.1c
Find the area of quadrilaterals using models.</td><td>Student finds the area of a quadrilateral.

Hector wants to make placemats for his mother’s table. He needs to know the area so that he can buy fabric for the placemats. This shows the size of a placemat. The width and length are labeled. Find the area of the placemat.
Area = length x width
 
</td><td>Concrete Understandings:
• Use square tiles to cover a rectangle.
• Count the number of tiles to determine the area.</td><td>Representation:
• Use formula to find area.
• Understand the following concepts and vocabulary: base, height, area, quadrilateral.</td></tr>
<tr><td>Measurement
MAFS.6.RP.1.AP.3c
Solve one-step real-world measurement problems involving whole number unit rates when given the unit rate (“Three inches of snow falls per hour, how much falls in six hours?”).</td><td>Student solves problem using rate.

It is raining hard today. The meteorologist said that rain is falling 2 inches per hour. It has rained for 3 hours so far. How much rain has fallen?
 
It is raining 2 inches per 1 hour.
 
How much rain has fallen in 3 hours?</td><td>Concrete Understandings:
• Multiply using concrete objects.
• Divide using concrete objects.
• Use a ratio to solve a measurement conversion problem.</td><td>Representation:
• Multiply whole numbers.
• Divide whole numbers.
• Use a pictorial representation of a ratio to solve problem.</td></tr>
</table>

Measurement MAFS.6.G.1.AP.1b Decompose complex shapes (polygon, trapezoid, and pentagon) into simple shapes (rectangles, squares, triangles) to measure area.	"To find the area of an irregular shape, you can draw a line to divide it into common shapes like rectangles and triangles. Draw a line on the shape below to divide it into two common shapes."  If the student makes an error, draw a line dividing the shape above into a rectangle and triangle for them. Give them the formulas below. "First, calculate the area of the rectangle. The formula for area of a rectangle is length times width." Pause and wait for student to perform skill. "Now, calculate the area of the triangle. The formula for area of a triangle is one half the base times the height." Pause and wait for student to perform skill. "Last, add the area of the rectangle to the area of the triangle to get the total area for the shape. Write your answer here." area of a rectangle ____ X ____ = ____ Length Width Area area of a triangle 1/2 X ____ X ____ = ____ Base Height Area ____ + ____ = ____ area of rectangle area of triangle total area of shape	**Concrete Understandings:** • Recognize simple shapes within a larger shape. • Identify the dimensions (base, height, length, width, etc.) of smaller shapes. • Multiply fractions and whole numbers.	**Representation:** • Given a picture, identify the dimensions of two-dimensional and three-dimensional shapes. • Understand the following concepts and vocabulary: polygon, trapezoid, pentagon, rectangles, squares, triangles, area.

Grade 8			
Access Point	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Geometry: MAFS.8.G.1.AP.4a Recognize congruent and similar figures.	"Now I want you to compare figures. If I say a figure is congruent, that means it is both the same shape and the same size. If I say a figure is similar, that means it is the same shape, but that it is a different size." Show the student the figures below. "Which shape is congruent with this one" Point to the one on the left. You can read the numbers to students if needed.  Show the student the figures below. "Which shape is similar with this one" Point to the one on the left. You can read the numbers to students if needed.  	**Concrete Understandings:** • Recognize corresponding points and sides in figures (e.g., match concrete examples of congruent shapes, match concrete examples of similar shapes).	**Representation:** • Understand the following concepts and vocabulary: figures, congruent, similar. • Describe circles, squares, rectangles, and triangles by telling about their shape, sides, lines, and angles.

Measurement: MAFS.8.G.3.AP.9a Using a calculator, apply the formula to find the volume of three-dimensional shapes (i.e., cubes, spheres, and cylinders).	"Here is a picture of a sphere. A sphere is a three-dimensional circle, like a ball. I want you to find the volume of the sphere. Remember, volume is the space inside an object. The formula for calculating the volume of a sphere is volume equals four thirds pi r cubed." Show the student the formula.  $V = 4/3\ \pi\ r^3$ http://3dshapes.org/3d-shapes-sphere-shape.html "Four thirds is the same thing as 1.33, so we can replace that in the formula." Show this formula. Student writes in 1.33 or shows teacher where to write it. "Remember that pi is 3.14, so let's replace pi with 3.14." Student writes in 3.14 or shows teacher where to write it. "Also, the radius is written as r. The radius of our sphere is 4. Write 4 in all three spaces for the radius." Student writes 4 in all three spaces or shows teacher where to write it. _____ = ___ x ___ x ___ x ___ x _____ Volume 4/3 π r r r "Now calculate the volume of the sphere."	**Concrete Understandings:** • Recognize attributes of a three-dimensional shape. • Multiply whole numbers, fractions, and decimals.	**Representation:** • Recognize that the volume of three-dimensional shapes can be found by finding the area of the base and multiplying that by the height. • Understand the following concepts and vocabulary: volume, cylinder, cone, height, radius, circumference, cube, sphere, side, pi.

4. What are Some Additional Activities That Can Promote Use of this Academic Concept in Real World Contexts?

-  How many feet of wallpaper border are needed for a bedroom wall that is 11 feet long and 9 feet wide?[11]
-  Mr. Cruz wants to get the best deal. Which freezer has the most cubic feet for the least amount of money? Freezer A has 25 cubic feet and costs $20 dollars per cubic foot. Freezer B has 16 cubic feet and costs $25 dollars per cubic foot.[12]
-  A house has a roof with the dimensions of 42ft by 24ft. If plywood comes in pieces that measure 8 feet by 4 feet, how many pieces of plywood are needed to cover the roof?[13]
-  If you have a plot of land, how many feet of fencing would it take to enclose it? How much corn could you plant on it?[14]
-  Brenda wants to paint her room. It measures 14 feet x 16 feet x 10 feet. One gallon of paint costs $20 and covers 250 square feet. The paint is sold only in 1 gallon cans. How much will it cost to paint the room?[15]
-  Mr. Lee wants to build a sandbox 5 feet long, 4 feet wide, and ½ foot high. What length of 6 inch boards will he need to surround the sandbox? How much of his yard will the sandbox cover? How much sand will he need to fill the sandbox? (First decide what you need to know, and then put your information into the formula and do the calculations.)[16]
-  How many square yards of carpet are needed to carpet a room that is 15ft by 25ft?[17]
-  You have a part-time job at a school. You need to buy enough grass seed to cover the school's soccer field. The field is 50 yards wide and 100 yards long. One bag will cover 5000 square feet. How many bags do you need?[18]

5. How Can I Further Promote College and Career Readiness when Teaching "Measurement and Geometry"?

Ideas for Promoting Career/ College Ready Outcomes

<u>Communicative competence:</u>
Students will increase their vocabulary to include concepts related to "measurement and geometry." In addition, they will be learning concepts such as: "around", "on", "in", "how many", "how much", "same", and "different."

[11] Bailey, R., Day, R., Frey, P., Howard, A. C., Hutchens, D. T., McClain, K., Moore-Harris, B. Glencoe/McGraw-Hill. (2004). *Mathematics: Applications and concepts*. New York: Glencoe/McGraw-Hill.
[12] Greenes, C. E. (2005). *Houghton Mifflin math: Grade 5*. Boston, MA: Houghton Mifflin Co.
[13] Bennett, J. M., & Holt, Rinehart, and Winston, inc. (2004). *Holt pre-algebra*. Orlando, Fla: Holt, Rinehart and Winston.
[14] Bennett, J. M., & Holt, Rinehart, and Winston, inc. (2004). *Holt pre-algebra*. Orlando, Fla: Holt, Rinehart and Winston.
[15] Bennett, J. M., & Holt, Rinehart, and Winston, inc. (2004). *Holt pre-algebra*. Orlando, Fla: Holt, Rinehart and Winston.
[16] Malestsky, E. M., & Andrews, A. G. (2004). *Harcourt math: Grade 4*. Orlando: Harcourt School Publishers.
[17] Larson, R., & McDougal, Littell. (2004). *McDougal Littell middle school math: Course 1*. Evanston, Ill: McDougal Littell.
[18] Larson, R., & McDougal, Littell. (2004). *McDougal Littell middle school math: Course 1*. Evanston, Ill: McDougal Littell.

Fluency in reading, writing, and math

Students will have opportunities to increase their numeracy and sight word fluency while participating in problem solving related to “perimeter, area, volume, and surface area” such as number recognition, counting, one-to-one correspondence, and reading concepts that include the use and understanding of prepositions.

Age appropriate social skills

Students will engage in peer groups to solve problems related to “measurement and geometry” that will provide practice on increasing reciprocal communication and age appropriate social interactions. For example, students might work together with their peers to find the surface area of a present to determine the amount of wrapping paper needed to cover it.

Independent work behaviors

By solving real life problems related to “measurement and geometry” students will improve work behaviors that could lead to employment such as landscaping, packaging, construction, and painting. When providing opportunities for real life problems leave some materials out and prompt/teach the students to determine who they should ask and what they should ask for to be able to solve the problem.

Skills in accessing support systems

At times, students will need to ask for assistance to complete activities related to “measurement and geometry” which will give them practice in accessing supports. Students will gain practice asking for tools such as talking calculators or a digital tape measure. They can ask a peer to complete the physical movements of the tasks they are not able to do themselves. Be sure to teach students to ask versus having items or supports automatically given to them.

6. How Do I Make Instruction on “Measurement and Geometry” Accessible to ALL the Students I Teach?

6.1 Teach Prerequisites and Basic Measurement and Geometry Skills Concurrently: Remember that students can continue to learn basic numeracy skills in the context of this grade level content.

Basic numeracy skills that can be worked on as a part of a lesson relating to perimeter, area, volume, and surface area:

- identify the figures: circle, rectangle, or triangle; then find the area, circumference or perimeter
- classify figures as 2- or 3-dimensional
- identify numbers while working with them in context of measurement and geometry
- identify the number of sides or faces of 2-D and 3-D figures
- count tiles, cubes, lines, and other units of measure
- learn to match numbers and symbols (=, +, *) to put an equation into a calculator while computing measurement and geometry equations

6.2 Incorporate Universal Design for Learning (UDL) in planning, and provide for additional Differentiated Instruction when Teaching Measurement and Geometry

Some examples of options for teaching Measurement and Geometry to students who may present instructional challenges due to:

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	**Physical Disability or Motor Differences (such as weakness or motor planning difficulty)**	**Extremely limited evidence of experience/ skill or motivation/ attention.**	**Lack of or extremely limited use of speech.**
Options for Representation	**Provide auditory options:** – Talking calculator when solving equations – Text-to-speech software or voice recordings to read aloud story problems – Single message sequence voice–output devices to count aloud – Captioning software that presents auditory information visually **Provide tactile options:** – Object cues, using - miniature objects or other tangible symbols to assist with problem comprehension and operations – Tactile equation mat – Create numbers and symbols out of tactile materials – Raise edges of shape using Velcro or puffy paint – Use a ruler with raised numbers, Braille representation, or paperclips/popsicle sticks indicating inch/cm marks – Use raised lines on figures (e.g., yarn; wiki sticks) – Make shapes out of materials with texture like carpet or Velcro – Use tiles with raised numbers or dots **Provide visual and manipulative options to scaffold representation of concepts:** – Color code equations and corresponding parts	**Reduce Physical Effort :** – Place materials on slant board or eye gaze array – Display flip chart, interactive white board or other teaching materials at student eye level – Count the tiles when determining area using a step- by-step process which progresses through numbers – Student can scan an array of possible options and use a switch to select the number to complete the equation template – Use computer representation of figures that can be manipulated with switch	**Illustrate through multiple media:** – Utilize interactive whiteboard – Incorporate interactive websites that provide nonlinguistic tools for exploring math concepts: Interactive 3-D shapes http://www.learner.org/interactives/geometry/ Illuminations http://illuminations.nctm.org/ActivitySearch.aspx Math Open Reference http://www.mathopenref.com/ There are many resources listed here: http://www.udlcenter.org/implementation/examples – Use materials that have an immediate effect (e.g., find area of CD and then play it) – Use a talking calculator to solve perimeter/area/ volume equation	**Provide customized display of information:** – Consistent model by utilizing modes of communication used by students (point to symbols representing concepts, operations) – Teacher model competent use of AAC during instruction

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/ attention.	Lack of or extremely limited use of speech.
	of calculator to support students correctly entering equations – Provide manipulatives for quantities, such as Cuisenaire rods or counting cubes – Provide empty containers with cubes and clearly define volume as the cubes inside the containers			
Options for Expression	**Vary the methods for response by:** – Student states answer or scans raised numbers to select correct answer; use voice output devices for student to select the correct answer o Provide manipulatives for student to respond or contribute to interaction – Teach a symbol that means "area" for student to use (e.g., four- 1x1 ceramic tiles glued onto foam board) or "volume" (e.g., four plastic cubes glued together) – Student states answer by selecting picture or symbol	**Provide options for responses/expression:** – Student selects numbers versus writing them; matches numerals and operation symbols to equation – Selection of correct answer is done after a model o Student points to each tile while teacher or peer counts aloud – Student answers "yes/no" questions regarding area after tiles have been counted aloud (e.g., 1,2,3,4. The area of this square is 4 square inches, is that correct?) – Matches the area to the correct number (matches 4 to 4) **Optimize access to tools/ alternatives for responding:** – Provide symbols, objects, manipulatives, and pictures for matching/ student responses	**Provide multimedia options for responses/expression:** – Allow the student to make selections by pointing to, gazing at, or selecting answers on the interactive white board – Utilize a switch or adapted computer mouse o Have student write answers with novel pencil or use a tablet computer – Determine area of "fun" objects (e.g., table with a party table cloth, cover of their favorite book)	**Provide options for modes of communication:** – Incorporate responses into student's AAC device or eye gaze array – Phrase questions so that they require a "yes/no" response, these can easily be answered using an eye gaze, head turn, two switches, etc. – Choose response by pointing to or selecting object or item o Use a blink response to count tiles or select answer – Count tiles/cubes out loud having student move in some voluntary way (e.g., nod head, tap hand, tap foot) to count along

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/ attention.	Lack of or extremely limited use of speech.
Options for Engagement	**Recruit interest by providing choices:** – Digital /talking representations, videos, talking calculators – Interactive websites **Increase personal relevance:** – Use items that are familiar and reinforcing to students – Incorporate high preference items into story problems, as well as student names **Provide tactile options for engagement:** – Use different types of textures to raise edges of box side or cardstock – Change the "feel" of a tile that has been counted to make it easier for the student to feel what has been counted and what has not	**Recruit interest by increasing personal relevance:** – Ensure that engaging and high preference content is visible and accessible to student – Use figures that are large enough to accommodate the movements that the student is able to make – Pair student with another student without a physical impairment and have them complete hands on activities together	**Recruit interest by providing choices:** – Digital /talking representations, videos, talking calculators **Provide options for sustaining effort and persistence:** – Break tasks down to maximize student attention – Token economy system that embeds equations (You have 2 Justin Bieber tokens. You need 5 total. How many more do you need to earn before you can listen to a song?) – Vary demands and materials to maintain interest – Assign the area, perimeter, and volume computations as a job task; student is "paid" 1 minute on computer for each one completed. **Increase personal relevance:** – Use items that are familiar and reinforcing to students. – Incorporate high preference items into story problems, as well as student names – Allow students to make posters for favorite sports team (middle MASSI)	**Recruit interest with modes of communication:** – Allow students to choose items or subjects that are relevant to them via AAC devices, symbols, or eye gaze array

Promoting Career and College Readiness	Standards for Mathematical Practice
1 Communicative Competence	1 Make sense of problems and persevere in solving them.
2 Fluency in reading, writing, and math	2 Reason abstractly and quantitatively.
3 Age appropriate social skills	3 Construct viable arguments and critique the reasoning of others.
4 Independent work behaviors	4 Model with mathematics
5 Skills in accessing support systems	5 Use appropriate tools strategically.
	6 Attend to precision.
	7 Look for and make use of structure.
	8 Look for and express regularity in repeated reasoning