

NCSC Curriculum Resource to Prepare Students for AA-AAS

Mathematics Content: Data Analysis

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Curriculum Resource to Prepare Students for AA-AAS

Mathematics Content: Data Analysis

The purposes of the Curriculum Resource Guides are:

- To provide guidance for teaching the Florida Standards to students with Significant Cognitive Disabilities (SWSCD) that both aligns with these standards and provides differentiation for individual student needs
- To provide examples for differentiating instruction for a wide range of SWSCD. These examples can be used in planning specific lessons, alternate assessment items, and professional development.
- To delineate the necessary skills and knowledge students need to acquire to master these indicators

1. What is “Data Analysis” and how is it taught in general education settings?

1a.1 The essential knowledge in this content area

Types of Displays of Data Taught in the Florida Standards

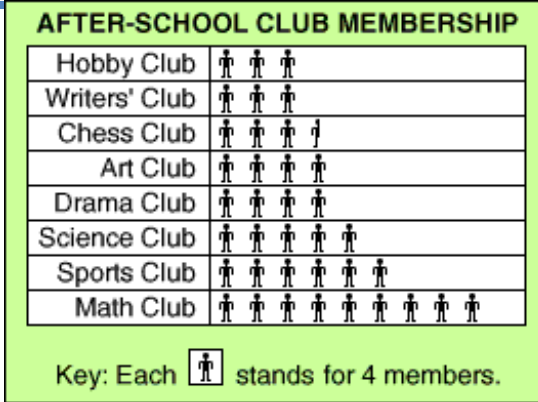
	Definition	Example																								
Tables	Tables are an organized way of displaying numerical data. They do not allow for visual analysis of data which can make it difficult to make inferences about the data displayed.	<table border="1"> <thead> <tr> <th colspan="3">Tourists to Australia (2006)</th> </tr> <tr> <th>Country of origin of visitors</th> <th>Number of visitors</th> <th>Average length of stay (nights)</th> </tr> </thead> <tbody> <tr> <td>Italy</td> <td>51 737</td> <td>42</td> </tr> <tr> <td>China</td> <td>308 452</td> <td>48</td> </tr> <tr> <td>United States of America</td> <td>456 084</td> <td>24</td> </tr> <tr> <td>United Kingdom</td> <td>734 244</td> <td>34</td> </tr> <tr> <td>Canada</td> <td>109 843</td> <td>42</td> </tr> <tr> <td>New Zealand</td> <td>1 075 797</td> <td>14</td> </tr> </tbody> </table> <p>http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2008/numeracy/NN_Data/NN_Data_s4b.htm</p>	Tourists to Australia (2006)			Country of origin of visitors	Number of visitors	Average length of stay (nights)	Italy	51 737	42	China	308 452	48	United States of America	456 084	24	United Kingdom	734 244	34	Canada	109 843	42	New Zealand	1 075 797	14
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Definition

Example

Pictograph

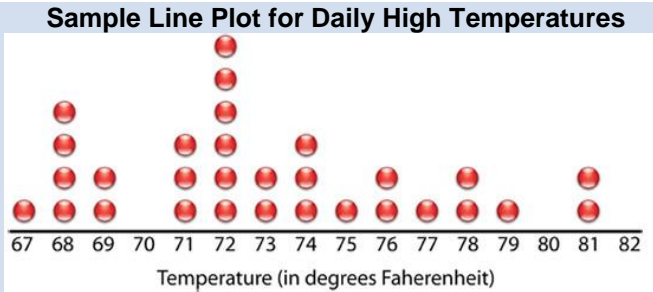
Pictographs display data using pictures symbols. Usually the pictures symbols represent a specific quantity. Data are usually displayed in rows or columns. Pictographs require a key to allow the reader to interpret the data.



http://www.asd4.org/curriculum/math/Math_ISAT_Word_Definitions_K-8.htm

Line Plot

A line plot is a method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.

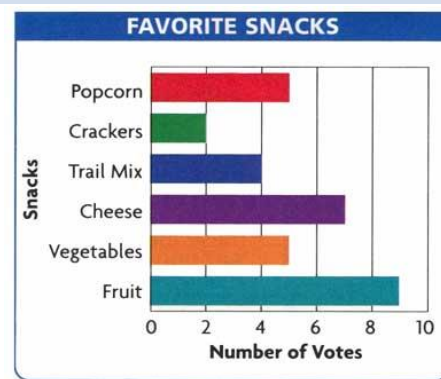


http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

<http://illuminations.nctm.org/LessonDetail.aspx?ID=L287>

Bar Graphs

Bar graphs are used to compare data. The data displayed is usually categorical (divided into categories). Bar graphs can be horizontal or vertical. The bars provide a visual display of the data that allow for comparisons to be made. When reading bar graphs, it is important to pay attention to the scale of the graph when interpreting data.



<http://www.future-edge.com/blackboard/MathM3E122/index.html>

Line Graphs

Line graphs are usually used to demonstrate changes in data over time. The horizontal axis usually represents the quantity and the vertical axis represents units of time. Data are plotted by using the comparison of two variables (bivariate data). Line graphs can be used to observe possible trends in data.



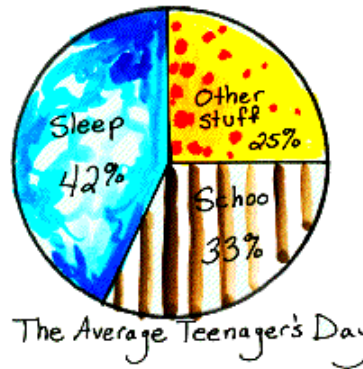
http://www.asd4.org/curriculum/math/Math_ISAT_Word_Definitions_K-8.htm

Definition

Example

Circle graphs (aka Pie chart)

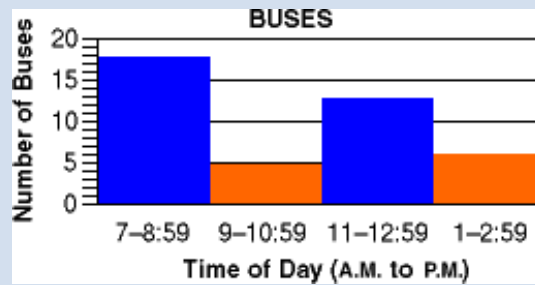
Circle graphs are used to demonstrate parts of a whole. The circle is divided into sectors, which represent the relative size of each part. Sometimes sectors represent percentages of a whole. The area of each sector in a circle graph is the same proportion to the whole circle as each item is to the total value in the table.



<http://www.themathlab.com/dictionary/pwords/pwords.htm>

Histograms

Histograms appear similar to bar charts. The main difference is that bars in histograms represent intervals of data and the data are numerical in nature (bars in bar graphs tend to represent categories). Due to the use of intervals, the bars in histograms touch each other to represent the continuous nature of the data.



http://www.asd4.org/curriculum/math/Math_ISAT_Word_Definitions_K-8.htm

Stem-and-Leaf plot

Stem-and-leaf plots organize the data in a way that displays the shape of the data (the way the data is distributed across the range). The data is organized by place value. The stem contains the largest place values and the leaves contain the numeral in the smallest place value. The leaves are usually displayed to the right of the stem. Another benefit to stem-and-leaf plots is that all individual data are displayed in the graph.

Points Scored in Basketball Games

Stem	Leaves
3	9
4	0 1 5 7 7 7 9
5	2 6 6 6

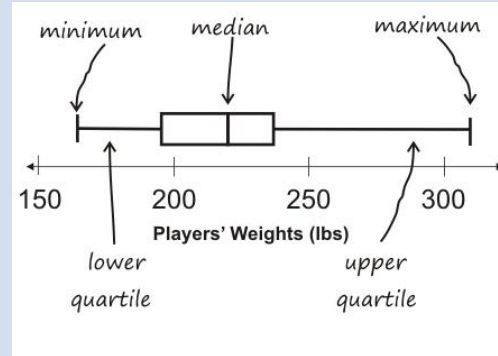
http://www.eduplace.com/math/mw/minv/6/minv_6c10.html

Definition

Example

Box plot
(aka box
and
whiskers
plot)

Box plots visually display the distribution of data. The box represents the middle 50% of the data. Data are divided into interquartile ranges (ranges that demonstrate 25% of data). The lower quartile (lower 25% data values) is demonstrated by the line extending from the left of the box (this line is called a whisker). The upper quartile (upper 25% data values) is demonstrated by a line extending from the right of the box (also called a whisker). The line inside the box represents the median value of the data set.

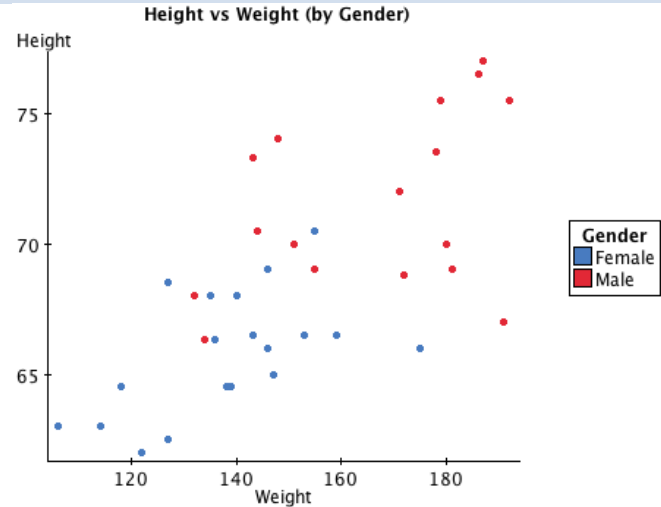


<http://bell7algebra.wikispaces.com/Box-and-Whisker+Plots>

Scatter plot

A scatterplot is a graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.

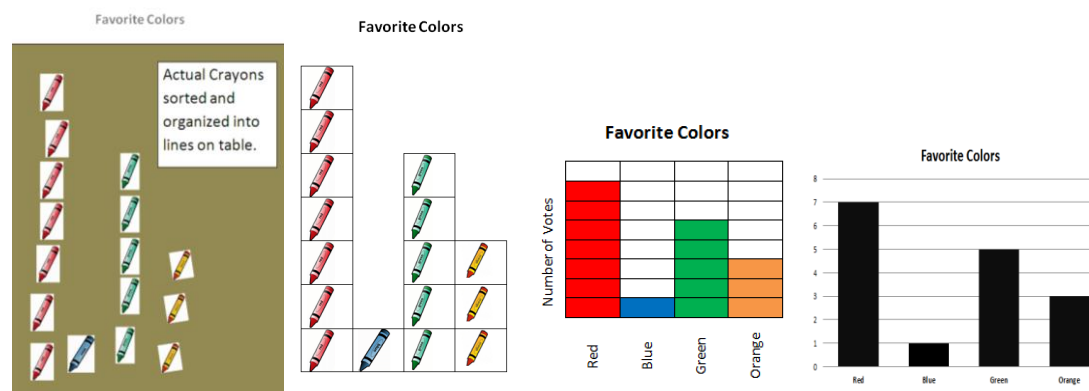
http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf



<http://www.statcrunch.com/5.0/viewresult.php?resid=834073>

Collect data and organize into bar graph

In general education settings, students are taught to pose a question, collect data, organize the data, and interpret the data. Children begin with using concrete objects as their “data.” They begin by classifying and sorting the items. Teachers can shape this into graphing behavior by having them sort the items into lines. Then students can begin to use representations of the objects (e.g., photos or line drawings) and place them onto graphs. This is followed by more advanced graphing which includes labeling the graph, shading in sections, developing a key or scale interval to represent amounts, etc. For example, students take votes on their classmates’ favorite colors. They get the following results: red – 7, blue – 1, green – 5, orange - 3. Then they are asked to graph the data. Below are examples of graphs moving from concrete to symbolic.

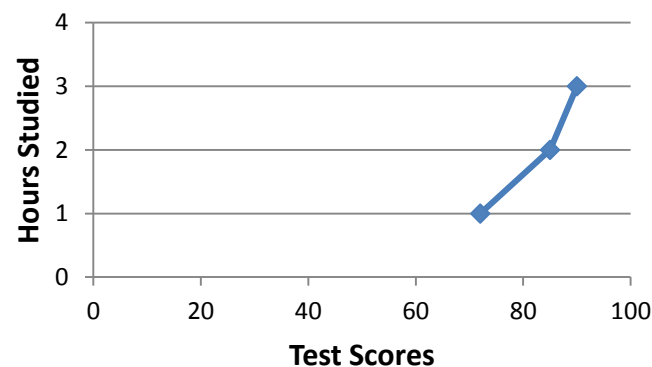
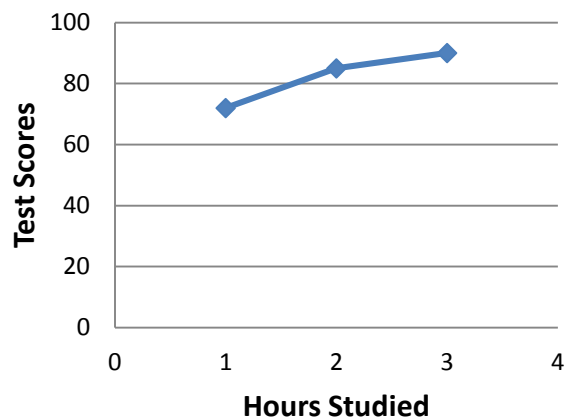


Concrete -----> Symbolic

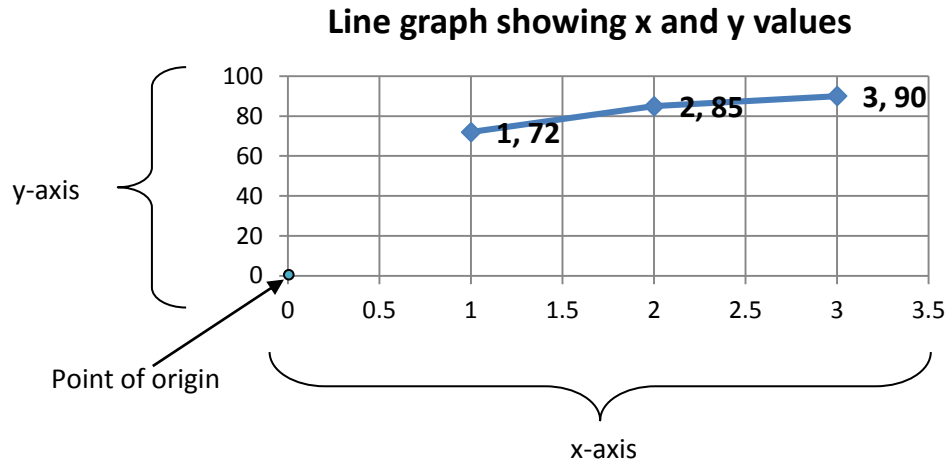
Using ordered pairs to graph given points

Ordered pairs are considered bivariate data. Bivariate data are data that consist of two variables such as miles per hour or numbers of hours studied and grades on the test. Line graphs displaying bivariate data help to show trends over time and enable students to make predictions/estimations by looking at points on the line between the plotted data and estimating the value of the variables represented. Creating line graphs requires students to utilize horizontal and vertical axes in order to plot the data points. This prepares students for graphing on coordinate planes.

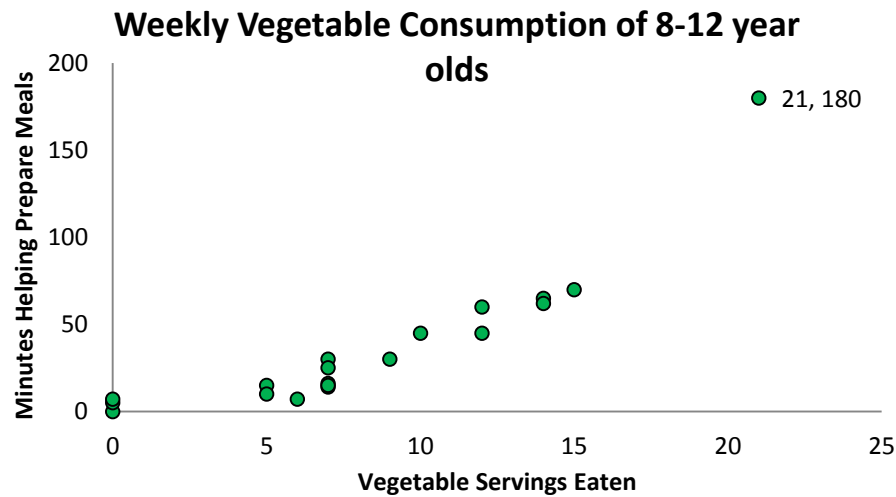
Below are two line graphs using the following bivariate data set: Several students were surveyed on how many hours they studied for the Algebra test. The average test score for students who studied 1 hour was 72%. The average for 2 hours was 85% and the average for 3 hours was 90%. Graph the data.



As shown above students may graph the data in two different ways based on their interpretation of the data and the axis where they assign the variables (test scores, hours studied). However, if all the students were told to assign hours studied to the x-axis and test scores on the y-axis or given the following ordered pairs (1, 72), (2, 85), (3, 90) the students should all create the same graph (as shown below). These ordered pairs demonstrate points on the graph corresponding with the x-axis (horizontal axis) and y-axis (vertical axis). The first number tells students how far to move away from the point of origin (zero on the graph or coordinate plane) across the horizontal axis. The second number tells students how far to move away from the point of origin up the vertical axis. So in the example below the first point on the graph is (1, 72). To plot this point you would move horizontally away from the origin towards 1, then move up vertically to approximately 72 and draw a point.



Bivariate data can also be graphed as scatter plots. A scatter plot is a graph of plotted points (i.e., ordered pairs) that show the relationship between two sets of data. In the example below, each dot represents one adolescent's vegetable consumption versus the number of minutes spent helping to prepare meals.



Students learn to analyze scatter plots to describe relationships between the two variables. Based on the plot above students would infer that the more time adolescents spend preparing meals the more likely they are to eat vegetables.

Summarizing data using measures of central tendency and variability

Data sets can be summarized in relation to measures of central tendency (the middle values of the data set). The three measures of central tendency are mean, median, and mode. See examples below using the following data set which compares the number of vegetable servings eaten weekly versus minutes spent weekly helping prepare meals of 19 adolescents:

Veg. Eaten	0	0	0	5	5	6	7	7	7	7	7	9	10	12	12	14	14	15	21
Min. Preparing	0	5	7	15	10	7	16	14	30	25	15	30	45	45	60	65	62	70	180

Mean, Median, and Mode

Mean is the sum of all the values divided by the total number of values. Mean of minutes preparing meals:

$$0+5+7+15+10+7+16+14+30+25+15+30+45+45+60+65+62+70+180= 701$$

$$701 \div 19 = 36.9 \text{ minutes}$$

$$\text{Mean number of minutes preparing meals} = 36.9 \text{ minutes}$$

Median is the middle number in the data set. To find the median, place the numbers in value order and find the middle number. Median of minutes preparing meals:

How data are presented:

0	5	7	15	10	7	16	14	30	25	15	30	45	45	60	65	62	70	180
---	---	---	----	----	---	----	----	----	----	----	----	----	----	----	----	----	----	-----

Data in value order:

0	5	7	7	10	14	15	15	16	25	30	30	45	45	60	62	65	70	180
---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----

Middle number:

0	5	7	7	10	14	15	15	16	25	30	30	45	45	60	62	65	70	180
---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----

$$\text{Median number of minutes preparing meals} = 25$$

*Note: if you have an even number of values, find the two middle values add them together and divide by 2 (e.g., middle numbers 24 and 30. $24 + 30 = 54$. $54 \div 2 = 27$. Median = 27)

Mode is the most often occurring value in the data set. To find the mode, place the numbers in value order and find the most often occurring number(s). Mode of minutes preparing meals:

Data in value order:

0	5	7	7	10	14	15	15	16	25	30	30	45	45	60	62	65	70	180
---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----

Mode(s):

0	5	7	7	10	14	15	15	16	25	30	30	45	45	60	62	65	70	180
---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----

The values 7, 15, 30, and 45 all appear twice in this data set. They are the modes.

Modes* of minutes preparing meals: 7, 15, 30, 45

*This data set would be considered multimodal (more than 2 modes; bimodal refers to two modes).

Variability (Range and Outliers)

Range

The range of a data set is the difference between the highest and lowest values in the set. Range of minutes preparing meals:

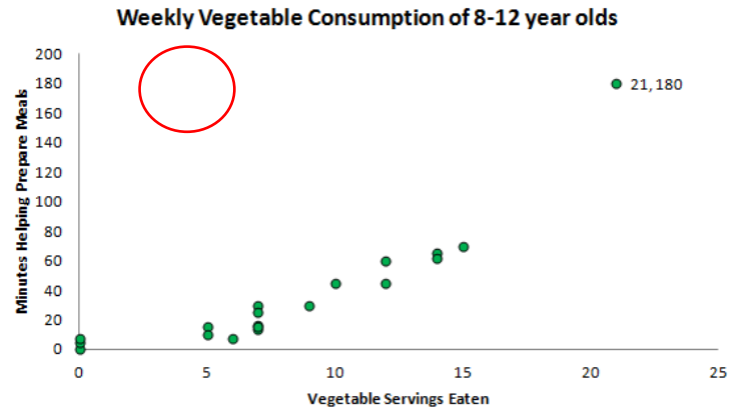
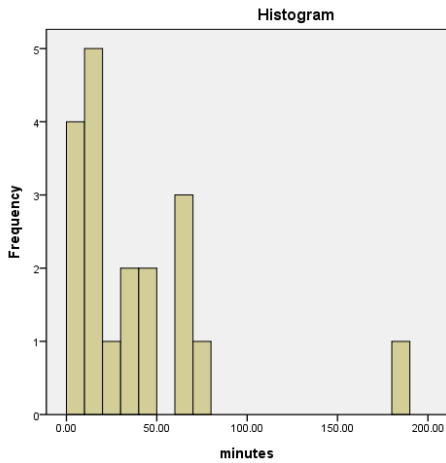
Highest – lowest = range

$$180 - 0 = 180$$

The range of minutes preparing meals is 180 minutes.

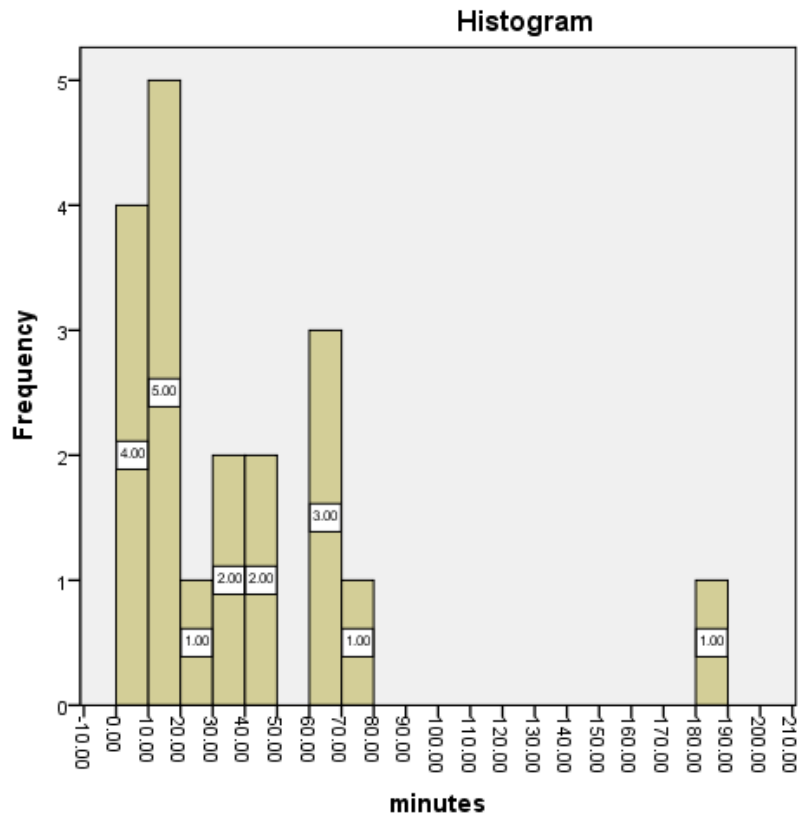
Outliers

An outlier is a data value that stands out from others in a set. Outliers are most easily determined by looking at a visual graph of the data. Below are two examples of viewing outliers (circled).



Summarizing data by creating histograms

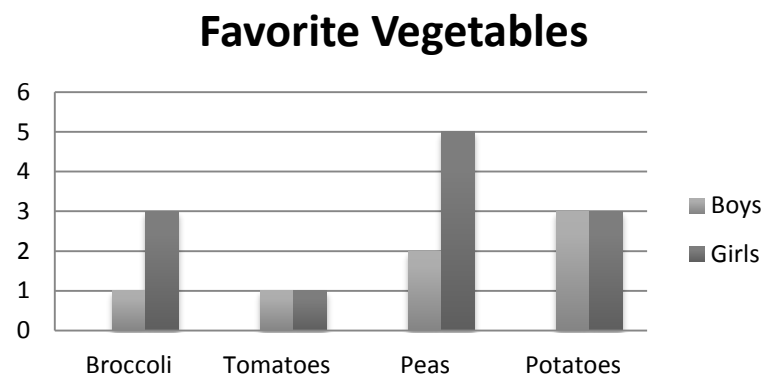
A histogram is a graphical display of data using bars of different heights. It is similar to a bar chart but a histogram groups data values into ranges and uses vertical columns to show the frequencies of the data values. The histogram below displays the number of minutes adolescents spend preparing meals per week.



According to this histogram the overall range of the data values is from 0 to 190. The data values are grouped in ranges of 10. This graph can be analyzed to interpret information about the adolescents in this sample duration of weekly meal preparation. For example, five students spent 10 to 20 minutes per week preparing meals. One student spent 70 to 80 minutes. Most students spent less than 1 hour preparing meals.

Analyzing graphs that include two samples or populations

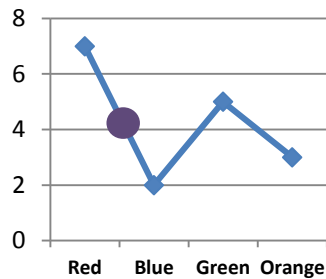
As students learn about measures of central tendency and variability they should be provided opportunities to apply their knowledge to a variety of graphs. Below is a bar graph that includes two samples or populations. This allows students to identify characteristics of each individual group as well as make comparisons between the two groups.



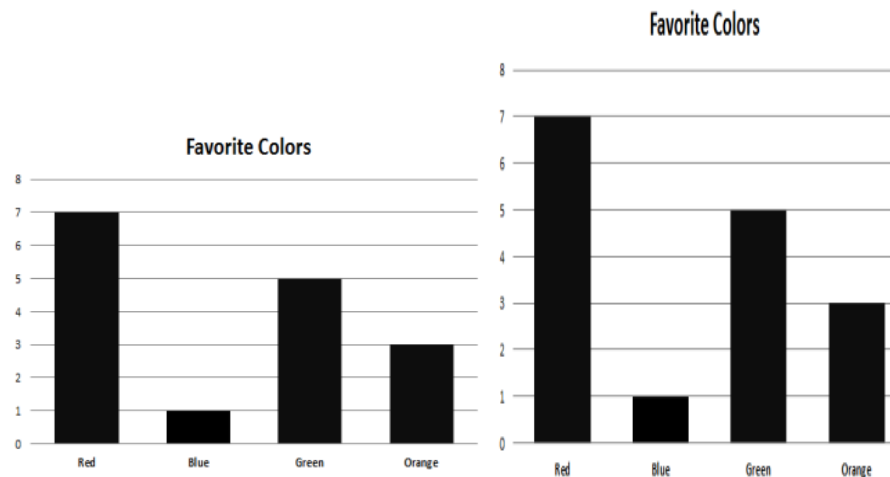
Examples of questions that could be asked about this data set: (a) Do more boys or girls like vegetables? [girls]; (b) What is the mode for favorite vegetables [most commonly voted for – peas]; (c) What is the difference between the number of boys who voted for broccoli versus girls? [2]; (d) What is the total number of students who voted for potatoes? [6]

1a.2 Common misunderstandings in this content area

- Students may use the wrong type of graph to display data. Below is an example of the correct use of a line graph instead of a bar graph. In this instance students should understand that all points on a line in a line graph should represent a value. What is the value of the midpoint (see purple dot) between red and blue? A student might answer “4”, but “4” of what? This data should have been represented using a bar graph. In order to avoid this misconception, students should be provided practice identifying types of graphs and their purpose. For example, bar graphs tend to display categorical data (data that can only be divided into categories such as colors), whereas line graphs display nominal data (data that can be displayed using numbers).



- Students may not attend to scale when comparing data in two graphs. See the example below. Both are displaying the same data set but one might appear to have more due to the height of the columns. To avoid this error students should be taught to attend to all the features of the graph (in this case, the y-axis values).



















- Students may have difficulty interpreting pictographs where each picture stands for an amount greater than one.











1a.3 Prior Knowledge/skills needed (can be taught concurrently)

- Number identification
- More/less
- One to one correspondence
- Same/different







2. What are some of the types of activities general educators will use to teach this skill?

2.1 Activities from General Education Resources
















- 4 1 3 Ask students to collect data (e.g., time students spend playing video games or types of video games played) and then have them select the correct graphical representation to display the data.
- 1 4 Have students collect graphs from a variety of sources (internet, magazines, etc.) and sort them by type.
- 4 3 Create a scaled pictograph representing classmates' favorite sport.¹
- 4 1 Use line graph which charts height and weight of an individual to answer questions about the pattern of her growth.¹
- 2 1 Give students data, such as responses to a survey about favorite dessert, and have them develop a graph with an appropriate scale to represent the data.²
- 1 3 1 Provide the same data set displayed in two different forms: Bar graph and circle graph. Ask students to compare and contrast the information obtained from each graph.³
- 2 4 Provide students with list of numbers and ask them to calculate the average.³

-  **4**  **3** Have students collect data about students in their school and compile statistics by determining the mean, median, and mode of student characteristics (e.g., height, age, library use, and distance from home to school).³
-  **4**  **1** Ask students to create a circle graph demonstrating how they spend their money.³
-  **1**  **4** Ask students to use the internet to find the number of different species of animals housed in five major zoos, and create a bar graph of the data set.⁴
-  **4**  **2** Give students a set of data and ask them to create a line graph to represent the data set provided.⁴
-  **4**  **2** Give students a set of data and ask them to create a stem-and-leaf plot to represent the data provided.⁴

Links across content areas

- Social Studies:
 -  **2**  **4** Have students research the average household income for all the states surrounding their own and then create a pictograph using a dollar bill to represent \$1,000. ¹
 -  **4**  **4** Create a bar graph displaying distance from where the student lives to major cities in the United States.¹
 -  **5**  **4** Using election results displaying votes by county have students to calculate the mean, median and mode number of votes for two competing candidates in 10 local counties.⁴
- Literature:



¹ Willoughby, S. S., & SRA/McGraw-Hill. (2003). *SRA math: Explorations and applications*. Worthington, Ohio: SRA/McGraw-Hill.

-  **1**  **2** Read *Help is on the way for charts and graphs* by Marilyn Berry.²
-  **1**  **2** Read *The Magic School Bus Inside a Beehive* by Joanna Cole and graph an estimate of the number of eggs the queen bee lays in one minute, two minutes, etc. and develop an appropriate scale to match the data.²
-  **3**  **2** Write several sentences up on the board that use the term average but communicate different meanings and discuss this with the class. Then explain and discuss the meaning of average when applied to mathematics.²
-  **2**  **2** Read *Peanut Butter* and have students graph the relationship between peanuts and amount in ounces of peanut butter.³
-  **3**  **2** Ask students to read three different types of genres and count the number of words per minute read over a period of three minutes. Have them graph the data and discuss why some genres may take more time to read than others.⁴
- Science:
 -  **4**  **4** Have students plant a seed and measure its growth over time and graph the data.²
 -  **4**  **4** Have students plant a new seed and an old seed and measure their growth over time, graph the data on the same chart, and compare the data between the two seeds.²
 -  **1** Have students collect and graph data showing the number of tornadoes per year in the United States for the last 5 years.⁴

² Willoughby, S. S., & SRA/McGraw-Hill. (2003). *SRA math: Explorations and applications*. Worthington, Ohio: SRA/McGraw-Hill.

³ Ibid.

- Music:







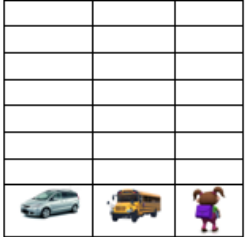












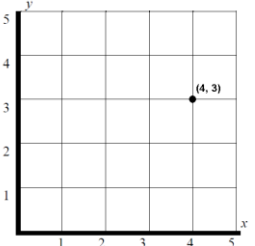
-  **5**  **3** Have students research the length of several popular songs and calculate the average length.²

3. What Access Points to the Florida Standards Are Addressed in Teaching “Data Analysis”?

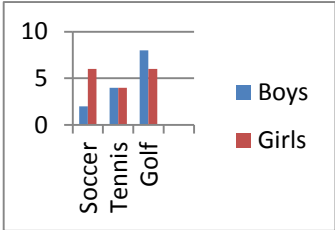
Grade Differentiation	Access Points
3 rd grade	MAFS.3.MD.2.AP.3a Collect data and organize into a picture or bar graph.
	MAFS.3.MD.2.AP.4b Organize measurement data into a line plot
	MAFS.3.MD.2.AP.3b Select the appropriate statement that compares the data representations based on a given graph (picture, bar, line plots).
5 th grade	MAFS.5.G.1.AP.1a Locate the x- and y-axis on a coordinate plane.
	MAFS.5.G.1.AP.1b Locate points on a coordinate plane.
	MAFS.5.G.1.AP.1c Graph ordered pairs (coordinates).
	MAFS.5.MD.1.AP.2a Collect and graph fractional data on a line plot (e.g., length of each person’s pencil in classroom, hours of exercise each week).
6 th grade	MAFS.6.SP.1.AP.1a Identify statistical questions and make a plan for data collection.
	MAFS.6.SP.2.AP.4a Display data on a line plot, such as dot plots, histograms or box plots.
	MAFS.6.SP.1.AP.3a Solve for mean of a given data set using whole numbers.










Grade Differentiation	Access Points
	<p>MAFS.6.SP.2.AP.5a Collect real-world data by surveying.</p> <p>MAFS.6.SP.1.AP.2a Find the range of a given data set</p> <p>MAFS.6.SP.1.AP.3b Explain or identify what the mode represents in a set of data.</p> <p>MAFS.6.SP.1.AP.2b Explain or identify what the mode represents in a set of data</p> <p>MAFS.6.SP.2.AP.5b Plot the data.</p> <p>MAFS.6.SP.2.AP.5c Define the mean, mode, and range of the data.</p>
7 th grade	<p>MAFS.7.SP.2.AP.4b Make or select an appropriate statement based upon two unequal data sets using measure of central tendency and shape of the distribution.</p> <p>MAFS.7.SP.1.AP.1a Survey a sample population to generate data that represents the total population.</p> <p>MAFS.7.SP.2.AP.4a Identify the range (difference), median (middle), mean (average), or mode (most frequent) of two sets of data.</p> <p>MAFS.7.SP.2.AP.3a Given graphed distributions of two sets of data, make statements comparing the two sets of data.</p>
8 th grade	<p>MAFS.8.SP.1.AP.1b Graph bivariate data using scatter plots and identify possible associations between the variables.</p>

Grade Differentiation	Access Points
	<p>MAFS.8.SP.1.AP.4a Analyze displays of bivariate data to develop or select appropriate claims about those data.</p>
Grades 9-12	<p>MAFS.912.S-ID.2.AP.5a Recognize associations and trends in data from a two-way table.</p>
	<p>MAFS.912.S-ID.1.AP.1a Complete a graph given the data, using dot plots, histograms or box plots.</p>
	<p>MAFS.912.S-ID.1.AP.4a Use descriptive stats like range, median, mode, mean and outliers/gaps to describe the data set.</p>
	<p>MAFS.912.S-ID.1.AP.2d Compare two or more different data sets using the center and spread of each.</p>
	<p>MAFS.912.S-ID.2.AP.6a Create a scatter plot from two quantitative variables.</p>
	<p>MAFS.912.S-ID.2.AP.6d Use algebraic methods and technology to fit a linear function to the data.</p>
	<p>MAFS.912.S-ID.2.AP.6b Make or select an appropriate statement(s) about findings</p>

Grade 3																
Access Point	Performance Example	Essential Understandings: Concrete Understandings and Representations														
Data: MAFS.3.MD.2.AP.3a Collect data and organize into a picture or bar graph.	Student reads a table and shades in a bar graph <table border="1" style="display: inline-table; margin-right: 20px;"> <thead> <tr> <th>Student</th> <th>Way to School</th> </tr> </thead> <tbody> <tr> <td>Napiqua</td> <td></td> </tr> <tr> <td>Arlo</td> <td></td> </tr> <tr> <td>Sydney</td> <td></td> </tr> <tr> <td>David</td> <td></td> </tr> <tr> <td>Jacob</td> <td></td> </tr> <tr> <td>Emilia</td> <td></td> </tr> </tbody> </table> 	Student	Way to School	Napiqua		Arlo		Sydney		David		Jacob		Emilia		Concrete Understandings: <ul style="list-style-type: none"> Identify data set based on a single attribute (e.g., pencils vs. markers,) Identify data set with more or less (e.g., this bar represents a set with more) Organize the data into a graph using objects (may have number symbols) Properly label graph (e.g. axes on bar graph) Representation: <ul style="list-style-type: none"> Identify data set with some number (e.g., bar graph representing 5 pencils) Identify a picture or bar graph
Student	Way to School															
Napiqua																
Arlo																
Sydney																
David																
Jacob																
Emilia																
Grade 5																
Access Point	Performance Example	Essential Understandings: Concrete Understandings and Representations														
Geometry: MAFS.5.G.1.AP.1c: Graph ordered pairs (coordinates).	 <p>Now it's time to plot points on this graph. Here is the x-axis (show student x-axis) and here is the y-axis. This point has already been plotted (point to 4, 3). This point is at 4, 3. Now it's your turn. Plot the following points (present each ordered pair one at a time and read it to the student).</p> <p style="text-align: center;">(2, 1) (1, 5)</p>	Concrete Understandings: <ul style="list-style-type: none"> Identify the x- and y- axis Identify the point of intersection Complete concrete graphing of points (e.g., put the straw up on 3 on the x-axis; put the next straw across on 2 on the y-axis. Put a chip there [for 3,2]) Understand first coordinate is horizontal, second coordinate vertical Representation: <ul style="list-style-type: none"> Understand concepts and vocabulary: coordinates, ordered pair, origin, axis, grid, point 														

Grade 6																														
Access Point	Performance Example	Essential Understandings: Concrete Understandings and Representations																												
<p>Data: MAFS.6.SP.2.AP.5c: Define the mean, mode, and range of the data.</p>	<p>Present students with the table and say “Mrs. Smith’s 6th grade class had a fundraiser to raise money for a class trip. Each student sold raffle tickets. This table shows how many raffle tickets each student sold.”</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Student</th> <th>Tickets Sold</th> </tr> </thead> <tbody> <tr><td>Frank</td><td>2</td></tr> <tr><td>Rita</td><td>1</td></tr> <tr><td>Derek</td><td>5</td></tr> <tr><td>Gabriel</td><td>4</td></tr> <tr><td>Francis</td><td>1</td></tr> <tr><td>Kamaria</td><td>3</td></tr> <tr><td>Steve</td><td>1</td></tr> </tbody> </table> <p>Show students the following response options. Say “The range of data is the highest number minus the lowest number. Which of these equations shows how to find the range?”</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>A. $5 + 1 = 6$</td> <td>B. $5 - 4 = 1$</td> <td>C. $5 - 1 = 4$</td> </tr> </table> <p>Show students the following response options. Say “The mode is the number that is repeated the most in the table. What is the mode of these data?”</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>A. 5</td> <td>B. 3</td> <td>C. 1</td> </tr> </table> <p>Show students the following response options. Say “You calculate the median by putting the numbers in order, smallest to largest, and then find the number in the middle. What is the median of these numbers?”</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>A. 2</td> <td>B. 5</td> <td>C. 1</td> </tr> </table> <p>Show students the following response options. Say “The mean of data is the average. To find the average, you add up all the numbers, which equals 17. Which equation shows the next step to find the mean?”</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>A. $17 / 1 = 17$</td> <td>B. $17 \times 2 = 34$</td> <td>C. $17 / 7 = 2.4$</td> </tr> </table>	Student	Tickets Sold	Frank	2	Rita	1	Derek	5	Gabriel	4	Francis	1	Kamaria	3	Steve	1	A. $5 + 1 = 6$	B. $5 - 4 = 1$	C. $5 - 1 = 4$	A. 5	B. 3	C. 1	A. 2	B. 5	C. 1	A. $17 / 1 = 17$	B. $17 \times 2 = 34$	C. $17 / 7 = 2.4$	<p>Concrete Understandings:</p> <ul style="list-style-type: none"> Identify the highest and lowest value in a data set given a number line and matching symbols. Arrange data from highest to lowest. Identify the representation (Plastic snap cubes, wiki sticks) of the mode. Use concrete materials to produce the mean (leveled plastic snap cubes). <p>Representation:</p> <ul style="list-style-type: none"> Identify the mode and the spread of the data using a line drawing of the distribution Calculate the mean using pre-slugged template of data points Understand concepts and vocabulary: mean, mode, spread of data
Student	Tickets Sold																													
Frank	2																													
Rita	1																													
Derek	5																													
Gabriel	4																													
Francis	1																													
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A. 2	B. 5	C. 1																												
A. $17 / 1 = 17$	B. $17 \times 2 = 34$	C. $17 / 7 = 2.4$																												

Grade 7					
Access Point	Performance Example	Essential Understandings: Concrete Understandings and Representations			
<p>Data:</p> <p>MAFS.7.SP.2.AP.4c: Make or select an appropriate statement based upon two unequal data sets using measure of central tendency and shape of the distribution.</p>	<p>Show the students the following bar graph. Say “The students in Mr. Kinnamore’s class took a poll of their favorite sports. The results are shown this graph. Boys are shown in blue and girls are shown in red. The sports are printed across the bottom and how many students liked that sport is shown on the left. For example: 6 girls like soccer and 2 boys like soccer.” Ask the students the following questions:</p>  <p>“Do more boys or girls like golf?”</p> <table border="1" data-bbox="436 732 1152 768"> <tr> <td>boys</td> <td>girls</td> <td>Same</td> </tr> </table> <p>Compare the mode for the favorite sports for boys and for girls. Remember, mode is the most commonly voted for sport. What is the difference number of boys who voted for the mode for boys and the number of girls who voted for the mode for girls?”</p>	boys	girls	Same	<p>Concrete Understandings:</p> <ul style="list-style-type: none"> Understand basic information from simple graphs (e.g., interpret a bar graph using the understanding that the taller column on a graph has a higher frequency, the shorter column on a graph has a lower frequency). Identify a representation of two bar graphs (one category apiece) as having greater or less frequency of members/events related to a single variable. (e.g., compare number of boys in soccer to girls in two graphs.) Make a comparison between two graphs <p>Representation:</p> <ul style="list-style-type: none"> Identify a pictorial representation of two bar graphs (one category apiece) as having greater or less frequency of members/events related to a single variable. (e.g., compare number of boys in soccer to girls in two graphs.)
boys	girls	Same			

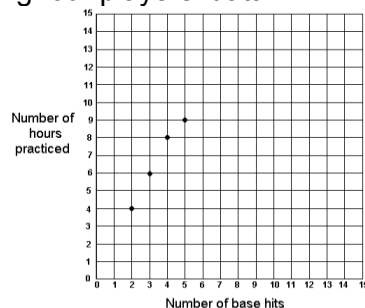
Access Point	Performance Example	Essential Understandings: Concrete Understandings and Representations																						
<p>Data: MAFS.8.SP.1.AP.4a: Analyze displays of bivariate data to develop or select appropriate claims about those data.</p>	<p>Show students the table and read the following: “The theater club is making posters to advertise their upcoming play. The theater director, Ms. Gowen recorded how long each student worked on making posters and how many posters they each made. Ms. Gowen put the data into a table, this column says the students’ names, this column says how many hours they worked on making posters, and this column shows how many posters each student made.” Read the table to the student if needed. Ask: “What is the relationship between the hours spent making posters and the number of posters each student made?”</p> <table border="1" data-bbox="520 646 1150 1157"> <thead> <tr> <th data-bbox="520 646 709 946">  Name </th> <th data-bbox="709 646 926 946">  Number of hours making posters </th> <th data-bbox="926 646 1150 946">  Number of posters made </th> </tr> </thead> <tbody> <tr> <td>Cosby</td> <td>1</td> <td>1</td> </tr> <tr> <td>Cruise</td> <td>2</td> <td>2</td> </tr> <tr> <td>Banks</td> <td>2</td> <td>3</td> </tr> <tr> <td>Hanks</td> <td>4</td> <td>6</td> </tr> <tr> <td>Roberts</td> <td>4</td> <td>7</td> </tr> <tr> <td>Ball</td> <td>5</td> <td>8</td> </tr> </tbody> </table>	 Name	 Number of hours making posters	 Number of posters made	Cosby	1	1	Cruise	2	2	Banks	2	3	Hanks	4	6	Roberts	4	7	Ball	5	8	<p>Concrete Understandings:</p> <ul style="list-style-type: none"> Match to same (a given association of a data set and three choices) Use graphic supports (e.g., highlighted transparency of an association) to identify the appropriate statement when given a relationship between two variables 	<p>Representation:</p> <ul style="list-style-type: none"> Explain the associations between the variables using supports (e.g., the selection of the highlighted transparency and make a statement) Understand concepts and vocabulary: variable, claim
 Name	 Number of hours making posters	 Number of posters made																						
Cosby	1	1																						
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Banks	2	3																						
Hanks	4	6																						
Roberts	4	7																						
Ball	5	8																						

Data:
MAFS.8.SP.1.AP.1b:
 Graph bivariate data using scatter plots and identify possible associations between the variables.

Show students the following table and say “This table shows the data for a baseball team. For each player, it shows the number of hours practicing in the batting cages and the number of base hits they made in a playoff run.”

Player’s Last Name	Number of hours practiced	Number of base hits
Schilling	4	2
Martinez	6	3
Garciparra	8	4
Nixon	9	5
Millar	10	6
Ramirez	11	8

“You are going to graph the number of hours practiced in the batting cages and number of base hits for these players. Number of hours practiced will be along the y-axis and the number of base hits will be along the x-axis. The shaded rows on the table have already been plotted; you need to plot the remaining four players’ data.”



Now show students the following response options and ask “Based on these data, what relationship do you see between hours practiced and number of base hits?” Read response options.

The more hours a player practices, the more hits he gets.

The more hours a player practices, the less hits he gets.

All the players have the same number of hits.

Concrete Understandings:

- Locate points on the x-axis and y-axis an adapted grid (not necessarily numeric).
- Match to same (a given association of a data set and three choices)
- Analyze scatter plots to check for: increasing or decreasing patterns in the data.

Representation:

- Graph a series of data points on a coordinate grid.
- Identify the associations between the variables using supports
- Understand concepts and vocabulary: best fit line, variable

High School																												
AP	Performance Example	Essential Understandings: Concrete Understandings and Representations																										
<p>Data:</p> <p>MAFS.912.S-ID.1.AP.4a: Use descriptive stats like range, median, mode, mean and outliers/gaps to describe the data set.</p>	<p>Show the students the following table and say: “This table shows the days each student in Mr. Wood’s class has missed at school this year. This column shows the student names and this column shows the number of days that student has missed.” Point to each column and show students. Read the first two rows to the students saying: “Look, Makayla has missed 0 days of school and Destiny has missed 1 day of school.” You may read the remainder of the table to the student if needed.</p> <table border="1" data-bbox="655 532 1060 917"> <thead> <tr> <th>Student</th> <th>Days Missed</th> </tr> </thead> <tbody> <tr> <td>Makayla</td> <td>0</td> </tr> <tr> <td>Destiny</td> <td>1</td> </tr> <tr> <td>Imani</td> <td>1</td> </tr> <tr> <td>William</td> <td>1</td> </tr> <tr> <td>Jaylen</td> <td>3</td> </tr> <tr> <td>Elijah</td> <td>5</td> </tr> <tr> <td>Sydney</td> <td>8</td> </tr> <tr> <td>Zion</td> <td>8</td> </tr> <tr> <td>Kevin</td> <td>9</td> </tr> </tbody> </table> <p>Present the word problem and response options (below). Read the word problem aloud to the student. The range is the difference between the fewest days of school missed and the most days of school missed. Which of these equations shows the range?</p> <table border="1" data-bbox="501 1084 1228 1133"> <tr> <td>$8 + 5 = 13$</td> <td>$9 - 8 = 1$</td> <td>$9 - 0 = 9$</td> </tr> </table> <p>Present the word problem and response options (below). Read the word problem aloud to the student. The mode is the number that occurs the most often. Which of these shows the mode for school days missed?</p> <table border="1" data-bbox="501 1300 1228 1344"> <tr> <td>1</td> <td>9</td> <td>8</td> </tr> </table> <p>Present the word problem and response options (below). Read the word problem aloud to the student. Once the numbers are put in order from smallest to</p>	Student	Days Missed	Makayla	0	Destiny	1	Imani	1	William	1	Jaylen	3	Elijah	5	Sydney	8	Zion	8	Kevin	9	$8 + 5 = 13$	$9 - 8 = 1$	$9 - 0 = 9$	1	9	8	<p>Concrete Understandings:</p> <ul style="list-style-type: none"> Given a scatter plot, identify outliers in the data set. Identify the highest and lowest value in a data set given a number line and matching symbols (concept of range). Identify the representation (Use Plastic snap cubes to represent the tally showing the number of occurrences) of the concept of mode. Identify the concept of median using concrete representations of data (create a bar graph with an odd number of bars using snap cubes; arrange from shortest to tallest; student place fingers on two outside towers, knock towers over and move inward until they reach the one middle tower left standing). Use concrete materials to produce the mean (leveled plastic snap cubes: using the same bar graph with snap cubes, re-arrange <p>Representation:</p> <ul style="list-style-type: none"> Understand concepts and vocabulary: median, mode, mean, outliers Identify the mode and the spread of the data using a line drawing of the distribution Calculate the mean using pre-slugged template of data points Order data set using numeric symbols
Student	Days Missed																											
Makayla	0																											
Destiny	1																											
Imani	1																											
William	1																											
Jaylen	3																											
Elijah	5																											
Sydney	8																											
Zion	8																											
Kevin	9																											
$8 + 5 = 13$	$9 - 8 = 1$	$9 - 0 = 9$																										
1	9	8																										

	<p>greatest, then you can find the median. The median is the middle number of the set of school days missed by each student. Which of these shows the median for school days missed?</p> <p><input type="text" value="1"/> <input type="text" value="3"/> <input type="text" value="9"/></p> <p>Present the word problem and response options (below). Read the word problem aloud to the student. The mean is the sum of the total school days missed divided by the number of students in the table. Which of these shows the mean for school days missed?</p> <p><input type="text" value="3"/> <input type="text" value="7"/> <input type="text" value="4"/> <input type="text"/></p>	<p>cubes into equal stacks).</p>	
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Data:
MAFS.912.S-ID.1.AP.1a
 Complete a graph given the data, using dot plots, histograms or box plots.

Show students the following table and say: “You are going to make a histogram. This table shows that data that will be in the histogram. It shows the students names on this side and the number of tickets sold on this side.” Point to each column as you explain. “Cassie sold 3 tickets, Erik sold 7 tickets.” Teachers can stop reading the table or read the entire table to the students if needed. “The ranges of tickets sold are listed below the table: 1-5 tickets, 6-10 tickets, 11-15 tickets, 16-20 tickets. Use those ranges on your histogram. Use this template to make your histogram. Remember to label both axes.”

Student	Tickets Sold
Cassie	3
Erik	7
Jackie	10
Andrew	5
Isaac	12
Tatiana	1
Bradley	3
Brendan	14
Elizabeth	8
Robert	2
Tommy	2
Michael	9
Kate	1
Billy	4
Justin	17
Jill	19

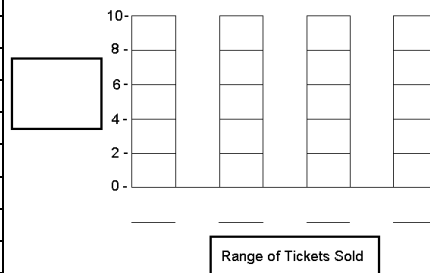
- Range of Tickets Sold:
- 1-5 tickets
 - 6-10 tickets
 - 11-15 tickets
 - 16-20 tickets

Concrete Understandings:













- Complete a graph
- Match the source of the values at the bottom of the x-axis with the appropriate category of the related data table.
- Describe the elements within a graph (e.g., in a box plot the line is the median, the line extending from each box is the lower and upper extreme, and the box shows the lower quartile and the upper quartile)

Representation:

- Complete the steps of the task analysis to complete a box plot
- Understand concepts and vocabulary: quartile, median, intervals, upper and lower extremes, box plot, histograms, dot plots



4. What are Some Additional Activities That Can Promote Use of this Academic Concept in Real World Contexts?

-  **3**  **5** Have students collect nutrition information on their favorite foods from several fast food restaurants and graph the amount of fat and total calories for each of the items. Ask them to identify the healthiest foods displayed.⁴
-  **4**  **3** Ask students to collect and graph the average temperatures of weather across the year in the area in which they live; then have them match the type of clothing they need to wear across the year.
-  **4**  **4** Have students graph their monthly allowance (or job earnings) and predict how long it would take for them to purchase a desired item. Extend this by having them choose an inexpensive item (such as a t-shirt) and a more expensive item (such as a Nintendo DS) and calculate the difference in time it would take to have the money to purchase these items.
-  **3**  **5** Ask students to research the amount of UVB and UVA sunlight that is emitted over time and compare this to what are considered harmful levels of each. Based on this information ask students how often they should reapply sunscreen and/or how long it is safe to stay out in direct sunlight.
-  **1**  **5** Have students set a goal time for running or walking two miles. Ask them to time themselves each time they run or walk two miles and predict how soon they will achieve their goal.⁴
-  **3**  **4** Get students to research the cost of living for the city in which they reside. Then have them research the median wages earned for 5 professions they may be interested in pursuing and compare these wages to the cost of living. Ask the students if they will be able to live comfortable on these wages? Will they need to work more than one job?

5. How Can I Further Promote College and Career Readiness when Teaching “Data Analysis”?

Ideas for Promoting Career/ College Ready Outcomes

Communicative competence:

Students will increase their vocabulary to include concepts related to “data analysis.” In addition, they will be learning concepts such as: more, less, most, least, same, different, average, about, and graph.

Fluency in reading, writing, and math

Students will have an opportunity to increase their numeracy and sight word fluency while participating in problem solving related to “data analysis” such as addition and division to determine mean and number identification to express mode or median. Writing numerals and developing

⁴ Bailey, R., Day, R., Frey, P., Howard, A. C., Hutchens, D. T., McClain, K., Moore-Harris, B. Glencoe/McGraw-Hill. (2004). *Mathematics: Applications and concepts*. New York: Glencoe/McGraw-Hill.
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questions about a data set of visual display of data. Reading and interpreting data displays including legends and labels on graphs.

Age appropriate social skills

Students will engage in peer groups to solve problems related to “data analysis” that will provide practice on increasing reciprocal communication and age appropriate social interactions. For example, students might work together with their peers to survey students in their school and then work together to display the data they collected into an appropriate graph.

Independent work behaviors

By solving real life problems related to “data analysis,” students will improve work behaviors that could lead to employment such as a data entry operator. When providing opportunities for real life problems leave some materials out and prompt/teach the students to determine who they should ask and what they should ask for to be able to solve the problem.

Skills in accessing support systems

At times, students will need to ask for assistance to complete activities related to “data analysis” which will give them practice in accessing supports. Students will gain practice asking for tools such as talking calculators, number lines, graphic organizers, and formulas. They can ask a peer to complete the physical movements of the tasks they are not able to do themselves. Be sure to teach students to ask versus having items or supports automatically given to them.

6. How Do I Make Instruction on “Data Analysis” Accessible to ALL the Students I Teach?

6.1 Teach Prerequisites and Basic Numeracy Skills Concurrently: Remember that students can continue to learn basic numeracy skills in the context of this grade level content.

Basic numeracy skills that can be worked on as a part of a lesson relating to equations:














- Number identification
- Counting
- One to one correspondence
- Addition
- Division
- Counting by twos (and other intervals of scale)

6.2 Incorporate UDL: Universal Design of Learning When Teaching Data Analysis

Some examples of options for teaching Data Analysis to students who may present instructional challenges due to:				
	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/attention.	Lack of or extremely limited use of speech.
Options for Representation	<p>Provide auditory options</p> <ul style="list-style-type: none"> Text-to-speech software or voice recordings to read aloud story problems Single message sequence voice–output devices to count aloud Captioning software that presents auditory information visually Provide student with tally counter that they can click for each count of the data value Use talking calculator <p>Provide tactile options:</p> <ul style="list-style-type: none"> Object cues, using miniature objects or other tangible symbols to assist with problem comprehension Create graphs out of tactile materials Use raised textured surfaces to demonstrate bars on graphs <p>Provide visual and manipulative options to scaffold representation of concepts:</p> <ul style="list-style-type: none"> Provide manipulatives for quantities, such a Cuisenaire rods or counting cubes Use Wiki sticks to raise grid lines Provide actual or miniature objects that represent each category or concept demonstrated in graphs Color code bars and data points with unit on x-axis Use numbers between 1-10 and provide number line which visually shows numbers getting larger. Use pictures for axis labels. 	<p>Reduce Physical Effort</p> <ul style="list-style-type: none"> Place materials on slant board or eye gaze array Display flip chart, interactive white board or other teaching materials at student eye level Student can scan an array of possible options and use a switch to select the quantity to complete the graph Use computer representation of figures that can be manipulated with switch Create large scale graphs on walls or floor so that students can easily navigate to or gesture towards components of the graphs 	<p>Illustrate through multiple media</p> <ul style="list-style-type: none"> Display data and graphs on the interactive whiteboard Incorporate interactive websites that provide nonlinguistic tools for exploring math concepts: <p>Illuminations http://illuminations.nctm.org/ActivitySearch.aspx</p> <p>Math Open Reference http://www.mathopenref.com/</p> <p>There are many resources listed here: http://www.udlcenter.org/implementation/examples</p> <ul style="list-style-type: none"> Have students create graph related to personal interests such as sports, music, movies, food, etc. Allow student to select topic for survey. Make activity hands-on. 	<p>Provide customized display of information</p> <ul style="list-style-type: none"> Consistently model by utilizing modes of communication used by students (point to symbols representing concepts, operations) Teacher model competent use of AAC during instruction

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/attention.	Lack of or extremely limited use of speech.
Options for Expression	<p>Vary the methods for response by:</p> <ul style="list-style-type: none"> – Student states answer or scans raised numbers to select correct answer – Provide manipulatives for student to respond or contribute to interaction – Student states answer by selecting picture or symbol – Use voice output devices for student to select the correct answer – Visually impaired student may dictate how to draw bars, order date, or graph linear equation – Student may also use talking calculator to solve for mean and range 	<p>Provide options for responses/expression:</p> <ul style="list-style-type: none"> – Student selects numbers versus writing them – Selection of correct answer is done after a model – Students identify values by matching numbers in data set to answer questions – Ask questions that allow them to answer “more”, “most”, “less”, or “least” instead of exact numeral values. <p>Optimize access to tools/ alternatives for responding</p> <ul style="list-style-type: none"> – Provide symbols, objects, manipulatives, and pictures for matching/ student responses – Let students use software to develop visual representations of the data they have created – Students may use speech-to-text software to input data. 	<p>Provide multimedia options for responses/expression:</p> <ul style="list-style-type: none"> – Allow the student to make selections by pointing to, gazing at, or selecting answers on the interactive white board – Utilize a switch or adapted computer mouse – Have student write answers with novel pencil or use a tablet computer – Use software that creates visually dynamic graphs to display and interpret data 	<p>Provide options for modes of communication:</p> <ul style="list-style-type: none"> – Incorporate responses into student’s AAC device or eye gaze array – Phrase questions so that they require a “yes/no” response, these can easily be answered using an eye gaze, head turn, two switches, etc. – Choose response by pointing to or selecting object or item – Use a blink response to count tiles or select answer – Count tiles/cubes out loud having student move in some voluntary way (e.g., nod head, tap hand, tap foot) to count along

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/attention.	Lack of or extremely limited use of speech.
Options for Engagement	<p>Recruit interest by providing choices:</p> <ul style="list-style-type: none"> – Digital /talking representations, videos, talking calculators – Interactive websites <p>Increase personal relevance:</p> <ul style="list-style-type: none"> – Use items that are familiar and reinforcing to students – Incorporate high preference items into story problems, as well as student names – Use items or activities of interest to represent within data sets <p>Provide tactile options for engagement:</p> <ul style="list-style-type: none"> – Use different types of textures to raise edges of box side or cardstock – Use a variety of interesting textures and colors to create graphs 	<p>Recruit interest by increasing personal relevance:</p> <ul style="list-style-type: none"> – Ensure that engaging and high preference content is visible and accessible to student – Use graphs that are large enough to accommodate the movements that the student is able to make – Pair student with another student without a physical impairment and have them complete hands on activities together – Use electronic or computerized voting software for students to gather and collect data 	<p>Recruit interest by providing choices:</p> <ul style="list-style-type: none"> – Digital /talking representations, videos, talking calculator – Allow students to use technology to calculate mean, median, mode, and range (e.g., Microsoft Excel®) <p>Provide options for sustaining effort and persistence</p> <ul style="list-style-type: none"> – Break tasks down to maximize student attention – Vary demands and materials to maintain interest – Provide reinforcement and incorporate interests as often as possible in data analysis activities. – Allow student to be in charge of sampling and data collection. <p>Increase personal relevance:</p> <ul style="list-style-type: none"> – Use items that are familiar and reinforcing to students. – Incorporate high preference items into story problems, as well as student names 	<p>Recruit interest with modes of communication:</p> <ul style="list-style-type: none"> – Allow students to choose items or subjects that are relevant to them via AAC devices, symbols, or eye gaze array

Promoting Career and College Readiness	Standards for Mathematical Practice
 1 Communicative Competence	 1 Make sense of problems and persevere in solving them.
 2 Fluency in reading, writing, and math	 2 Reason abstractly and quantitatively.
 3 Age appropriate social skills	 3 Construct viable arguments and critique the reasoning of others.
 4 Independent work behaviors	 4 Model with mathematics
 5 Skills in accessing support systems	 5 Use appropriate tools strategically.
	 6 Attend to precision.
	 7 Look for and make use of structure.
	 8 Look for and express regularity in repeated reasoning